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THE DOUBLE PASS METHOD FOR MORE PRECISE DETERMINATIONS OF LONGI--ETC(U)

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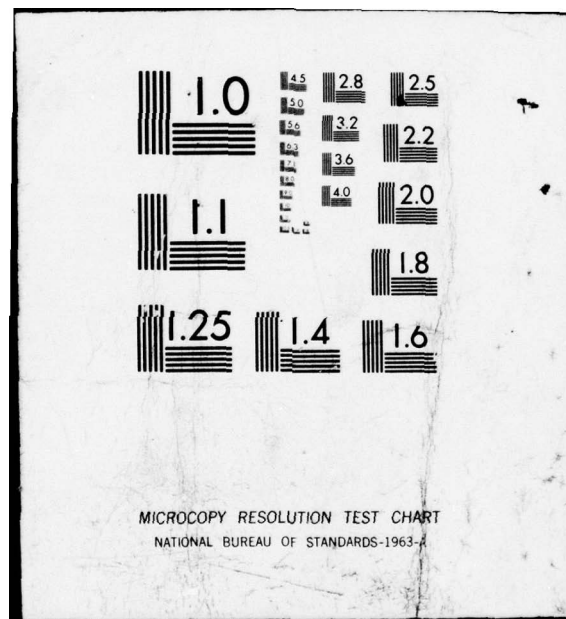
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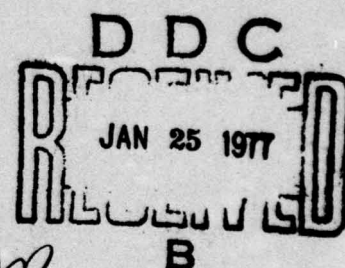
THE DOUBLE PASS METHOD FOR MORE PRECISE  
DETERMINATIONS OF LONGITUDE AND GEOIDAL HEIGHT  
USING THE TRANSIT SYSTEM OF NAVIGATIONAL SATELLITES

By  
EDUARD BERG

JULY 1975

Prepared for  
OFFICE OF NAVAL RESEARCH  
under Contract N00014-75-C-0209

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Date: 30 July 1975



## ABSTRACT

Using the Stansell (1970) elevation correction for navigational satellites together with the double pass method of Anderle (1971) and Greely (1971) but for individual navigational satellites on two consecutive passes, it is shown that uncertainties in longitude position can be reduced to the order of 10 to 18 m for land stations with just a few observations, and less than 4 m for dock-side ship stations taken over a period of a week. It is also shown that the method allows identification of satellites not giving reliable data and permits conclusions to be reached as to the cause of an apparent error dependence on surface elevation noted above 1000 m in the Andean region of Colombia, South America, reported by Woollard and Thompson (1974). An additional advantage of the double pass method brought out is that it allows geoidal heights to be determined for remote islands with the same degree of reliability as their position and comparable to the best, continental determinations of geoidal height using standard geodetic measurements.

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## INTRODUCTION

Hawaii Institute of Geophysics Report HIG-74-1 by Daugherty (Part I) and Woollard and Thompson (Part II) deals with the accuracy of satellite determined positions at various port sites around the Pacific and in the Andean region of Colombia, South America, and their difference from map coordinate positions.

As Daugherty in Part I (hereafter referred to as Daugherty) brings out the fact that successive passes by different satellites with the ship at a fixed position for periods of a few days to two weeks indicated that erratic values were obtained, there is some question as to how much value can be attached to a single pass satellite-determined position when a ship is underway. Woollard and Thompson in Part II (hereafter referred to as Woollard and Thompson) bring out a different type of problem; namely systematic differences in longitude for East- and West-passing satellites that appeared to be elevation-dependent at elevations above 1000 m. These data have been reexamined as to the causes of the discrepancies noted.

By way of review, Daugherty used a simple arithmetic averaging of the data obtained on successive passes of each satellite recorded by using a MX/702/hp satellite navigation system on University of Hawaii ships in 1970, 1971 and 1972 at eight port sites distributed throughout the central and south Pacific. Daugherty also averaged the results obtained for each individual satellite and computed the standard deviation of each pass from the mean as well as that of the mean values.

A summary of Daugherty's results with the number of observations at each site which varied from 6 to 153 are given in Table 1. It will be noted that two series of observations were made at Suva with significant differences in the apparent reliability of the longitude position as a result of an apparent error in initialized antenna height for the first series. In the present study the effect of the error (20 m) is evaluated and the data for Suva used to demonstrate the effectiveness of the double pass method for determining position and antenna height (geoidal height) at sea level sites.

Because of time restrictions, the land data reported by Woollard and Thompson were much more limited in terms of the number of satellite passes recorded at each site. The apparent ground elevation dependence of position, particularly longitude reported for East- and West-passing satellites, is brought out in Table 2 (from Woollard and Thompson, their Table 2).

Although Woollard and Thompson showed that if an empirical correction for the tangent of the satellite elevation angle were applied, reasonable agreement in position could be obtained with available maps, principally the ONC scale 1:1,000,000 series, the discrepancies at high elevations still exceeded those found in general for sea-level sites.

It was with the above datum sets that the writer started the investigation here reported. As part of the investigation, the reliability of maps



available in Colombia was also investigated since different map series for the same area indicated differences in coordinates for the same locations.

The principal differences in the writer's approach from that used by Daugherty and that used by Woollard and Thompson was in the application of: (1) the Stansell (1970) correction curve for antenna height, which while similar to the tangent correction incorporated by Woollard and Thompson for their corrected data, also appears to incorporate tropospheric refraction effects, and (2) the double pass method of Anderle (1971) and Greely (1971), published as U. S. Naval Weapons Laboratories technical reports that came to the writer's attention in an article by Schultz et al. (1975) after most of the work for this report was completed.

As the writer's interest in this problem was generated through his participation in Project Nariño, a seismic crustal study in Colombia which was depending on the satellite defined positions reported by Woollard and Thompson for the location of the land shot and recording sites, most of the report is devoted to this series of measurements and the reliability of the maps used in connection with Project Nariño.

Because of the large sample of data available for Suva and the fact that two different series of observations had been made with different initialized antenna heights, these data were also studied to see whether the same approach as applied to the Colombia data would significantly reduce the uncertainty in position found by Daugherty. As will be shown, this was indeed the case, and from the study it became apparent that the satellite data offers a direct method for determining geoidal height having the same degree of reliability as that of the derived longitude position.

In presenting this report the internal agreement between the two sets of maps used on Project Nariño will be discussed first. These were the Instituto Geografico "Agustin Codazzi" (IGAC) maps on a scale of 1:25,000 and maps prepared by the Texas Oil Company (Texaco) on a scale of 100,000. Both were based on the Bogota datum. The reading accuracy for comparisons between the two sets of maps was about 0.015'. As will be shown, discrepancies between the two sets of maps appear to lie within the accuracy with which positions could be read from the Texaco maps. On the whole, agreement was good except in Buena Ventura-Valle area, where a consistent discrepancy of about 0.6' in latitude was found to characterize part of the Texaco map.

The second section is a discussion of the double pass method for determining longitude and height using the Stansell (1970) elevation error curve for satellite elevation. The third section presents the results for the redetermination of positions in Colombia using the Stansell (1970) satellite elevation correction and the double pass method. In the fourth section the same methodology is applied to the large body of data available for Suva. As will be seen, this last indicates that positioning in longitude and height is better than 8 m for a daily mean, better than 4 m for a weekly mean, and about 2 m if a two-week mean is used, without earth- or ocean-tidal corrections.

## IGAC AND TEXACO MAP COMPARISON

Both the IGAC map and Texaco map coordinate systems are based on a gauss conformal projection with the origin at the National Observatory in Bogota, Colombia, at  $4^{\circ}35'56''570N$  and  $47^{\circ}04'51''300W$  and  $X = 1000N$  and  $Y = 1000E$ . Most of the available IGAC maps were on a 1:25,000 scale and the Texaco maps were on a 1:100,000 scale (with a  $3^{\circ}$  E-W offset). A computer program used here to transform the X, Y coordinates to latitude and longitude is based on J. H. Addison's program which was kindly supplied by J. F. Gettrust, now at the Australian National University in Canberra, formerly at University of Wisconsin, and adapted for the University of Hawaii computer by A. Kam. The constants used in the program are:  $A = 6378388$  (radius) and  $ES = 0.00672267$  (eccentricity<sup>2</sup>).

### Coordinate transformation

A check was provided by reading the x and y coordinates of the map corners of some Texaco maps (where  $\lambda$ ,  $\phi$  were also given) and computing  $\lambda$ ,  $\phi$  from x, y. These calculations are presented in Table 3. The C: line gives the map designation (such as L-1) and the geographical coordinates in degrees-minutes as indicated on the map; the following lines gives the x and y coordinates measured, and the calculated geographical coordinates in degrees-minutes, N latitude and W longitude. The standard deviation from the indicated coordinates is 0!013 in the N-S and 0!0076 in the E-W direction; these values include the measuring error and also the drafting inaccuracies (reduced to map: 0.24 mm NS and 0.14 mm EW).

We can therefore conclude that the computer program does not contain any gross errors.

### Inter-map comparison

To test the agreement between the IGAC and Texaco maps, several areas were selected for comparison. The selection was based on the proximity of the areas to the seismic refraction lines, satellite navigation fix points, and availability of maps. In each area a number of easily identifiable features were used for comparison points, such as river-bends, or road-river or railroad intersections. Comparisons for the areas are given in Table 4. Each point comparison consists of four printout lines:

Line 1: gives the area (at only the first mention)  
BUENA VENTURA-VALLE, IGAC map designation 260-III-C,  
and Texaco map designation J-4;

Line 2: IGAC map input x, y; output degrees, minutes N, W;

Line 3: Texaco input x, y; output degrees, minutes N, W; and

Line 4: difference in output  
degrees, minutes N, W.

The difference in line 4 is positive for the IGAC position being to the north and to the west of the Texaco position.

In general, the use of small river-bends as comparison points gave the best results. On the Texaco map, roads at times appeared to have been smoothed out; or just one side of the road located in its correct position with the other drawn parallel to it. The comparisons were relatively poor in the southern area of the Buena Ventura-Valle area since the available maps contained few good comparison points. However, serious discrepancies do exist. Some part of the NW corner of the J-4 Texaco map must have been shifted, as can be seen by the 0!6 southward shift for the north part and a 0.74 westward shift for the south part with respect to the IGAC map. The shift is clearly marked on the J-4 map, where a southbound road ends, with a "?" sign, some 1 km NE of the bend in the Buena Ventura road (heading EW) instead of joining it as it does on IGAC map 260-III-C. (Coordinates of the Buena Ventura road-bend  $x = 921.78$ ,  $y = 678.68$ ; that is the fifth point in the Buena Ventura area in Table 4.)

Table 5 summarizes the results of Table 4 by area. It indicates the N-S and E-W offsets for each area and the standard deviation of a single observation. Most of the offsets noted are close to one's ability to measure features correctly and to the accuracy of the drawing of the Texaco map as reflected by the standard deviations of about 0!015 (or near 28 m). Other than in the Buena Ventura area, on the eastern flank of the Cordillera (Mocoa) some 0!06 N-S and E-W offsets appear that are larger than the standard deviation of a single observation based on four points.

In conclusion, there are serious offsets (probably internal to the Texaco map) in the Buena Ventura area and a small one west of Mocoa. Except for those in the Buena Ventura area, none of the other small discrepancies should affect the positions of seismic refraction stations with respect to the geographical coordinates from either set of map coordinates.



## DOUBLE PASS METHOD APPLICATION TO LONGITUDE AND HEIGHT DETERMINATION OF THE SATELLITE NAVIGATOR

### Elevation angle correction

Large differences (500 m or more) appeared when the satellite navigation positions adopted by Woollard and Thompson were compared with the known map positions, especially for Tumaco and Laguna la Cocha. The question arose as to whether the tangent correction applied was valid. To test this, the data for Laguna la Cocha longitudes were plotted as a function of the tangent of the elevation angles (Fig. 1). As seen, the lines fitted to the E-passing satellites and the W-passing satellites do not intercept the zero elevation line at a common longitude. The resulting gap is of the order of 900 m. The computer program used by Woollard and Thompson (taking the least squares fit of all points) obviously provided a position somewhere in the gap as did their graphical solution using uncorrected data. The tangent correction, while clearly a step in the right direction on the basis of the model used by Woollard and Thompson to explain the cause of the discrepancies observed in longitude position as being an error in antenna height incorporated in the satellite software program, was not the complete answer. The answer itself became apparent from a paper by Stansell (1970) provided the writer by Dr. John Rose.

In this paper, Stansell (1970), shows a graph (his Fig. 11) which gives the fix error (in longitude) as a function of satellite elevation angle  $K$  that results from the error in the antenna height estimate (initialized antenna height). If  $f(K)$  is the fix error (in n. miles) resulting from a 1-m height error, then a dH-meter height error will result in a dH  $f(K)$  n. mile error in longitude.

If a number of satellite passes are recorded, each having elevation angle  $K_i$  and fix  $L_i$ , and data points for satellite pass no.  $i$  are plotted in a rectangular coordinate system at points  $y_i = f(K_i)$ ,  $x_i = L_i$  with no other noise present, then the points  $y_i$ ,  $x_i$  will fall on a straight line for which the slope is  $\frac{dy}{dx} = \frac{1}{dH}$ . The inverse slope will give the antenna height error and the value for  $L$  at  $y = 0 = f(K=0)$  will be the correct longitude position.

Since the input data were not known and no computer programs were available for calculating Stansell's (1970) graph, values were carefully measured from the Stansell diagram (given for changes in  $L$  for 100-foot errors in antenna height) and recalculated for the position changes that would result from a 1-m antenna height change. These values for  $10^\circ$  increments of elevation angle were plotted on a large graph and a smooth curve was drawn, from which the final values were read. (The approximate values for  $f(K)$  and  $K$  are listed in Table 6 and also shown in Figure 9.) Position changes that would result from a similar correction curve given

in the Magnavox 706 manual are slightly smaller than the ones derived from the Stansell curve. Therefore, antenna height changes based on the Magnavox curve would be slightly larger than the ones used here.

In Figure 2, a plot of  $f(K_1)$  versus  $L_1$  (the longitude fixes) for Laguna la Cocha is shown and the inverse of the slope defined indicates an elevation error of some 2000 m. Correcting the longitudes for an error of  $dH = 2110$  m and plotting again the reduced  $L_1$  versus  $f(K_1)$  values results in a remaining spread of some 0.2 min or nearly 0.2 n. miles (neglecting  $\cos$  (latitude)) (Fig. 3).

As the direction of the satellite navigator-computed longitude is the opposite from the direction of the passing satellite if the initialized antenna height is too high (that is toward the west for an east-passing satellite), or toward the satellite if the initialized height is too low, it is clear that the satellite navigator program (for Laguna la Cocha and other high-elevation stations) used a different (too elevated) antenna height than the one punched in by the equipment operator (N. J. Thompson). This will be discussed in more detail later.

#### Double pass method

As can be seen from Figure 3 the spread in the longitude positions can be reduced further (to 0.01 min or 20 m) if the data are restricted to height corrected values for individual satellites for which there are two consecutive passes. In the plot the value of  $f(K)$  for the elevation angle is the y coordinate and the corrected longitude the x coordinate with E-passing satellites plotted positive for  $f(K) = 0$  and W-passing satellites plotted negative. The intersection of the resulting lines of position with  $f(K) = 0$  defining the more precise position. When 64 double passes were analyzed for Suva, the standard deviation of a single such determination was found to be about 20 m in longitude and about the same in height (Tables 12 and 13). Part of this standard deviation seems to be due to differences in individual satellites, notably no. 64 in 1973 in Colombia (the same as no. 65 in 1971 in Suva) which shows a lower antenna height (Figs. 3, 16, and 18). In passing it should be noted that in general, the longitude position data for the double passes in Colombia in 1973 seem to indicate less spread than those recorded on board the KANA KEOKI in Suva in 1971. The lower accuracy for Suva is probably due to effects of the ship's motion with wave and tidal action in the harbor and perhaps to better satellite orbital parameter programming in 1973.

Note that no corrections have been applied for the tropospheric refraction index, for which the satellite navigator computer assumes an average value.

## COLOMBIA SATELLITE NAVIGATOR POSITIONS

Observation points--IGAC map positions

Woollard and Thompson presented the satellite navigation data together with site descriptions and compared them to positions on the 1:1,000,000-scale ONC L26 aeronautical chart. This chart, the only one covering all sites on the same projection and datum, was, however, inadequate for precise determinations of location because of the scale. Therefore, Thompson, who made the field observations in Colombia, and the writer reviewed site positions where possible on the IGAC maps with scales of 1:25,000, 1:10,000, and in one case, 1:2,000. Longitudes and latitudes were then computed from the x, y coordinates using the program supplied by G. F. Gettrust. The determined positions and elevations are given in Table 8. However, even better accuracy could be achieved if the sites could be resurveyed, especially the four for which photographs of the antenna positions are available. For convenience, the original satellite positions and heights from Woollard and Thompson are presented in Table 7. As it appears that the site elevations (surface elevation plus geoid height) were not used consistently in their report, the antenna height and geoid height as initialized in the field were taken directly from Thompson's original book and added as two additional columns in Table 7.

The estimated accuracies of x, y, and height as presented in Table 8 (and based on available photographs, scale of the maps used, and discussions with Thompson) with respect to the maps are as follows:

<u>Site</u>	<u>(N-S) x (in m)</u>	<u>(E-W) y (in m)</u>	<u>Height (m)</u>
Buga	$\pm 10$	$\pm 20$	$\pm 10$
Popayan I	$\pm 2$	$\pm 3$	$\pm 3$
Laguna la Cocha	$\pm 30?$	$\pm 20?$	$\pm 20?$
Tumaco	$\pm 15$	$\pm 15$	$\pm 2$

However, an accuracy of the order of 20 to 50 cm could be obtained if the sites were to be surveyed and based on the available pictures of the antenna positions.

Magnavox 706 height initialization error

The satellite longitude fix versus elevation angle function  $f(K)$  for the Colombia positions are presented in Figures 2 through 7. As Figures 2 and 3 indicate, a very large antenna height correction at Laguna la Cocha is required to reduce the data by the double pass method to the final mean longitude of  $77^{\circ}08'8050 \pm 0'0033$  ( $\sigma_M$ ). Since Cali (Fig. 5) and Popayan I (Fig. 6) also show large height corrections, an operator error for the height initialization is excluded.



The best explanation that appears reasonable for the capricious height initialization of the Magnavox internal computer will be discussed relative to Laguna la Cocha. As shown by Figures 2 and 3, the computer-initialized height is too high by  $2110 + (+7-27-153)/3 = 2052$  m. This means that the computer was working with an initial height somewhere above 2800 m plus 2052 m, or over 4800 m, if the site altitude of 2808 m is approximately correct. This could happen as follows (borne out also by the Cali and Popayan data): the computer dislikes altitudes over 999 (or 1024?) m. When an elevation over that height is initialized, the computer doubles the corresponding decimal (1000) or binary digit (1024). The data are too few and the uncertainty in the absolute value of the Stansell correction too large to determine which, but it probably is the binary. The result then is the following:

$$\begin{aligned} \text{Operator H} &= 2808 = 2048 + 760 \\ \text{Computer H} &= 2 \times 2048 + 760 = 4856 \\ \text{Concluded H} &= \text{Computer H} - \text{height correction} \\ &= 4856 - 2052 = 2804 \pm 60 \text{ m } (\sigma_M). \end{aligned}$$

Until the uncertainty in computer shift of 24 m/1000 m at elevations higher than 1000 m is resolved, the height data for Laguna la Cocha (48-m shift) and Popayan and Cali (24-m shift) should not be used without putting these possible errors in evidence. The Bogota data are inconsistent, probably because of interference problems, and have not been considered reliable for the double pass method.

#### Colombia satellite positions--comparison with IGAC coordinates

The satellite-determined coordinates and the IGAC map coordinates are compared in Table 9. In this table the satellite latitudes are those determined by Woollard and Thompson using intersecting lines of position for each satellite, except for Tumaco, for which an arithmetic mean value was used. The satellite longitudes are the values obtained here from the double pass method and are derived from the  $f(K)$  "0" crossings in Figures 2 through 7. The  $\sigma_M$  errors reflect the internal consistency of the data. The purpose of analyzing the Suva shipboard-fixed station data in the next section of this report is to substantiate the small errors found by applying the double pass method in particular to the Colombia data. All the Colombia satellite-determined longitudes are probably within 10 m, and similarly the heights for Buga, Tumaco, and Buena Ventura probably within better than 20 m. The height error evaluation for Cali, Popayan I, and Laguna la Cocha will depend on the Magnavox 706 programming error and, because of the large corrections involved, on the accuracy of reading the Stansell elevation angle correction. The height correction for Tumaco was obtained by best-fit combining satellites no. 41, 53, and 63 in pairs and determining the slope of the corresponding lines (Fig. 7). Satellite no. 64 was not used, since height determination from no. 64 seems distinctly different, as can be seen for Laguna la Cocha (Fig. 3) and the corresponding determinations in Suva

(satellite no. 65) (see Table 13 and Figs. 16 and 18). However, longitude positions for no. 64 do not deviate from those obtained by the double pass method from the other satellites. The no. 64 (in 1973) Colombia is the same satellite designated no. 65 (in 1971) in Suva.

So far no IGAC map position could be obtained for Cali and Buena Ventura. The position at Buena Ventura given in Table 9 is the one Thompson determined in the field at Buena Ventura from navigation chart HO 1781 (Woollard and Thompson's Table 1, p. 34). It does not seem sufficiently accurate for comparison of the map offsets, nor consistent with the differences found for the other sites.

As Table 9 shows, there exists a systematic offset between IGAC map- and satellite-determined latitudes and longitudes; so the satellite fix is between 0.100 and 0.180 min to the south and between 0.150 and 0.190 min to the east of the map position. The offset is well outside the error limits that can be assigned either to the map or to the satellite position accuracy, and probably relates to the Bogota datum.

#### Height relation to gravity

Although Woollard and Thompson showed that anomalous gravity could be related qualitatively to uncertainties in position, no quantitative relations were established. In the present study, and excluding the data for satellite no. 64, the difference between map height and satellite-determined height (the latter height including geoid undulations) was found to show a systematic relation to the regional simple Bouguer gravity anomaly of about 2 mgal per m height difference (see Table 9, last two columns, and Fig. 14), zero Bouguer anomaly corresponding to  $\approx 32$  m which is close approximation of the geoidal height where it reaches a maximum value in western Colombia.

# RELIABILITY OF POSITION DATA FOR LONGITUDE AND HEIGHT, SUVA 1971

An important question to be answered was whether or not the Colombia longitude and heights were as accurate as the internal consistency of the data suggested by their  $\sigma_M$ .

## Conventional arithmetic mean longitude and latitude

Since many longer series of observations were reported by Daugherty, any particular series at a fixed position could have been analyzed by the double pass method. However, C. Marsh (Hawaii Institute of Geophysics) suggested using the Suva data because he found that two different heights had been initialized. This circumstance, not reported by Daugherty, promised to provide a particularly good opportunity to check the elevation correction. In addition, a considerable number of passes had been recorded that were not reported by Daugherty, especially during the first few days, where initialized antenna height (including the geoid height) was 75 m. All available passes are presented in Table 10A, and all available passes where the initialized height was 54 m are listed in Table 11A. According to Daugherty, the information contained in the standard fix output of the HP2114 computer used was based on the MAPS-H-70356 program aboard the R/V KANA KEOKI in 1971. The parameters relating to the movement of the receiver were deleted and the format of the remaining quantities was changed as shown below:

- (1) DATE (DATE): The Julian Day Number representing the consecutive numbering of the days in the year;
- (2) TIME (GMT): The Greenwich Mean Time of the position fix in hours and minutes;
- (3) SATELLITES (SAT): Five near-polar-orbit navigation satellites were available for observation. A two-digit numbering code for satellites, related to the orbital semi-major axis in kilometers, was adopted. The correspondence to the satellite number is indicated below.

SAT	SATELLITE NUMBER
42	30120
54	30140
63	30180
64	30130
65	30190



The numbering code (left column) is higher by one unit than that used for the Colombia data by Woollard and Thompson, so that SAT 42 (Suva 1971) = SAT 41 (Colombia 1973) and so forth.

- (4) ELEVATION ANGLE (ELEV): Vertical angle, measured in degrees, of the satellite at the point of closest approach to the observing station;
- (5) GEOMETRY (GEOM): Indicates the satellite travel is either North (N) or South (S) bound, to the East (E) or West (W) of the observer;
- (6) LATITUDE (LATITUDE): Latitude of the observation point on the satellite datum measured in degrees, minutes, and seconds from the equator;
- (7) LONGITUDE (LONGITUDE): Longitude of the observation point on the satellite datum measured in degrees, minutes, and seconds from the Greenwich Meridian to the station;
- (8) ITERATIONS (IT): Number of iterations required for the program to converge on the positional fix;
- (9) DOPPLER COUNTS (CTS): Number of 24-sec interval counts used in the computation of the position fix;
- (10) DOPPLER COUNT SEQUENCE (CTSQ): Number of balanced (symmetric) 24-sec counts about the point of closest approach.
- (11) The arithmetic mean position in longitude and latitude was calculated for each of the two tables separately by retaining only those passes for which the following criteria were met:
  - (1)  $15^{\circ} \leq \text{ELEV} \leq 75^{\circ}$       elevation angle
  - (2)  $\text{IT} \leq 4$       iterations
  - (3)  $\text{Dev} < 10.0 \text{ arc sec}$       position deviation

The deviations from the mean latitude and longitude are also given in the Tables 10A, 11A, as well as the rejection criteria used (last three columns). The mean latitude and longitude together with their statistical reliability are given in Tables 10B and 11B, and the mean values for each satellite separately were presented in Tables 10C and 11C.

### Double pass--longitude and height results

All double passes (e.g. two consecutive passes of the same satellite) in Table 10 and 11 have been analyzed for longitudinal position and receiver height. Figures 8 through 13 show the  $f(K_1)$  versus  $L_1$  data for 12 consecutive 1971 days (203 to 214) for Suva. H 75 or H 54 indicates the initialized antenna height. Satellites are identified by their number (i.e., 42), direction of travel (S or N), and the chronological order of the center time of the two consecutive passes: 1 or 2--to 9 (on day 204). The longitude determined from the double pass is the position where the connecting line between the two passes crosses the x-axis; the inverse slope of the line, corrected for latitude, gives the antenna height correction.

It is clear from the crossings of the x-axis that one can easily distinguish good days (like 203 and 209) from bad days (like 210), and that bad double pass positions like 63S5 (at  $178^{\circ}25''.5800E$ ) on day 203 can be rejected. Tables 12 and 13 give longitudes and heights (e.g. H + correction) for all retained double passes, averages, standard deviations, and standard deviations of the mean, ordered by N- or S-bound satellites or by days, and for all satellites and days taken together.

It should be noted that for Suva, E is in the positive X direction on Figures 11 through 16, whereas E is in the negative X direction for Figures 1 through 7 pertaining to the Colombia longitudes. As a consequence, the antenna height correction changes sign between the two sets of figures for the same apparent slope.

Figures 15 through 18 are graphical summaries of the contents of Table 12 and 13. The deviation of the daily mean longitude from the total mean value only once exceeds 8 m (Table 12, Fig. 15), whereas the daily mean heights show somewhat larger variations (Table 13, Fig. 16), mainly due to the lower height determined from the mean of satellite 65. This is the same satellite (no. 64 Colombia) that showed a much lower height for Laguna la Cocha (Fig. 3) two years later. The lower height is also clearly borne out in Figure 16, where not even one height determined from no. 65 (Suva) (identified by +) reaches the mean, as determined by all double passes. The mean height determined from no. 65 of  $32.3 \pm 3.5$  m deviated by more than 26 m from the mean of the other four satellites of  $58.7 \pm 2.4$  m. The cause of this rather significant discrepancy is not known, but as Table 12 and Figure 18 indicate, it does not seem to affect the longitude observations. In Figure 18, the mean longitude and the mean height together with the  $2\sigma$  error-length are indicated for each satellite, and the positions marked S- or N- are the corresponding values for the southbound or the northbound orbits. The mean of all the satellite longitudes and the mean height without satellite no. 65 are indicated by the dashed lines together with the  $2\sigma$  error of the mean.

Finally, the writer attempted to detect any possible variations as a function of time of day that might depend on changes in ionospheric or tropospheric conditions. In Figure 17 the 64 double pass longitudes are shown as a function of the time of day. Values that have been rejected from visual inspection on a particular date are surrounded by a square. The two rejected double passes, closer to the mean than some of the non-rejected ones, both clearly were outside the remaining seven positions on day 204 (Fig. 8, Sat 64S2 and 64N7) and probably would have been retained on another date. The data seem to show a perhaps slightly tighter clustering between 14 and 18 hours GMT, which are the local early morning hours (from 2 to 6 a.m.).



## STANDARD ERROR--COMPARISON

In the following, discussion will be limited to longitude data, since no attempt was made to correct the latitude data. As was to be expected, the accuracy of the double pass longitude determination is considerably higher than that of the routinely used arithmetic averaging especially in the case where the antenna height was incorrectly initialized. The extreme example is represented by the longitudes determined for Laguna la Cocha (Table 7), where the  $\sigma$  for a single observation is 1'.41 (min of arc) when the arithmetic mean is taken, and only 0'.0047 for the double pass method. Even the more realistic comparison of the  $\sigma$  of the mean yields 0'.408 for the former versus 0'.0033 for the latter method, approximately two orders of magnitude better.

Arithmetic mean versus double pass method

Returning to the Suva data, we see that the set with the erroneously initialized height of 75 m shows a standard deviation for a single observation of (1.9 sec or) 0'.032 as compared to (1.5 sec or) 0'.025 (Tables 10B, 11B) for the set with the nearly correct antenna height of 54 m. In order to make a comparison with the double pass method, the preceding values should be divided by  $\sqrt{2}$  since in the latter method each position is determined from two consecutive passes. Therefore, even in the best possible condition of nearly correct antenna height initialization, the  $\sigma$  (two observations) is  $0'.025/\sqrt{2} = 0'.018$  as compared to 0'.011 for the double pass method (Table 12). The  $\sigma$  of the mean also would be comparably smaller since a total of 137 passes were accepted for the arithmetic mean method (see "NSD" in Table 10B and in Table 11B) versus 64 double passes (128 single passes). The improvement is about 1.6. However, as Table 10B shows (for antenna height error of about 20 m), the  $\sigma$  (single observation) increases to (1.9 sec or) 0'.032 or  $0'.032/\sqrt{2} = 0'.022$ , which results in an improvement of 2.0 for the double pass method.

Double pass method: fixed shipboard stations versus fixed land stations

The difference in accuracy between the shipboard data and the land-based data (in longitude) is somewhat more difficult to evaluate, since no single common set of passes is available and data from a single land station in Colombia have at the most three double passes. As discussed earlier, the results of all accepted double passes for Suva (Table 12) indicate a  $\sigma$  (single ship observation) of 0'.0111 or 19.6 m. However, in addition, some double pass positions were rejected. To evaluate the standard deviation for the land sites the following expression was used:

$$\sigma \text{ (single observ.)} = \left( \sum_i \sum_j \Delta x_{ij}^2 / (n-1) \right)^{1/2},$$

where  $\Delta X_{ij}$  is the deviation of the  $i^{\text{th}}$  station's  $j^{\text{th}}$  observations from the mean position of the  $i^{\text{th}}$  station; and  $n = 12$  is the total number of double pass positions obtained (for Buga, Cali, Popayan I, Laguna la Cocha and Buena Ventura; see Figs. 3 through 7). The value thus obtained of  $\sigma$  (single land observ.) = 0'.0045 (or 8.35 m) is less than half of that for the shipboard data. It should be emphasized strongly that none of the double passes recorded in Colombia had to be rejected.

Possible explanations of the difference in accuracy are:

- (1) ship motion near the docking site; and
- (2) the satellite orbital programming and the navigator programs used in 1973 in Colombia were more accurate than those used during 1971 in Suva.

## CONCLUSIONS

(1) Except for maps from the Buena Ventura area, the IGAC and TEXACO maps agree with sufficient accuracy to allow the determination of the seismic refraction stations to a common datum during project Nariño in 1973.

(2) For all Colombia sites for which IGAC map coordinates are available, the satellite navigation positions are to the South (by 0'.103 to 0'.180) and East (by 0'.152 to 0'.191) of the map positions. The combined (map position and satellite-navigator fix) longitude error should be smaller than 30 m and could be improved to less than 10 m if the sites were surveyed. The standard longitude deviation for a single double pass observation was 8.4 m.

Due to some error, probably internal, of the satellite navigator computer, elevations are reliable only for Buga, Tumaco, and Buena Ventura and are probably reliable with 20 m or less. Elevations for the remaining three sites should be used with caution.

(3) The shipborne, fixed-point satellite navigation determinations of longitude and height show a standard deviation for a single double pass observation of less than 20 m for all five operational satellites combined. However, significant differences for the height have been observed for satellite 65 when compared with the others. The improvement in longitude position is at least a factor of 1.6 over that obtained by the usual arithmetic averaging and increases with the antenna height error. For example, for the Suva longitude the improvement was a factor of 2 for an antenna height error of around 20 m. The standard deviation of the mean in Suva was less than 2.5 m in either longitude or height over a total of 12 days of recording time.

This accuracy suggests that many more harbor sites can be located with respect to the satellite navigation datum within 8 m for each coordinate from a single day's observations, and the local geoid height can be obtained as well, both a considerable advantage over the arithmetic averaging method.

(4) Differences in accuracy are noticed among individual satellites (at least for the Suva data). Whereas the  $\sigma$  (single) = 0'.0111 for all satellites combined, the  $\sigma$  (single) for no. 64 was only 0'.0061 and that for no. 63 was 0'.0092. Also, lower deviations were observed for northbound than for southbound orbits of individual satellites (exception: no. 42). In the extreme case of no. 64,  $\sigma$  (single N-bound) = 0'.0037 versus  $\sigma$  (single S-bound) = 0'.0089. One observation of concern was that satellite 65 indicated an antenna height lower by more than 26 m than those of the other satellites used.

In conclusion it appears that the satellite data can be used to determine geodetic coordinates (longitude, latitude and height) on a



worldwide basis with a reliability equal or close to that obtained by other more costly surveying methods. Bender (1974), for example indicates that there are very few locations in the world, and these limited to continental areas, where geodetic coordinates have been established with a reliability of 2 m.

Finally it should be pointed out that the question of the corrections applied and this report itself probably never would have been made if the Magnavox 706 satellite navigator had calculated the fixes with the initialized antenna heights instead of what it probably did: used a height that resulted from upward shift of one binary digit at elevations above 1024 m.

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## Addendum

After this report was completed, Mr. Willian Unger, University of Wisconsin, having received a preliminary copy, commented on (written communication, October 24, 1975) and sent calculations transforming the map coordinates from the Bogota datum to those for the NWL-9D\* coordinate system used.

The following table gives the resulting differences in latitude and longitude (in minutes of arc), as calculated by Unger from:

Map Coordinates from Table 9 for Stations Buga, Popayan I,  
Laguna la Cocha and Tumaco

NWL-9D Minus SAT-NAV Positions  
(in minutes of arc)

	$\Delta$ LAT	$\Delta$ LONG
Buga	+ 0.0071	- 0.0014
Popayan I	- 0.0273	+ 0.0077
Laguna la Cocha	- 0.0720	- 0.0298
Tumaco	+ 0.0061	- 0.0089

Besides the Laguna la Cocha differences, for which the map positions could not be established as well as for the other stations (error estimates with ? in Table 9), only the Popayan I latitude is outside the combined estimated map position error plus the  $\sigma_M$  of the SAT-NAV determination. However, if the arithmetic mean value of the SAT-NAV latitude is used (Woollard and Thompson's Table 4, p. 48) the  $\Delta$  latitude for Popayan I is - 0.0068 and within the combined errors.

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\*The NWL-9D system is the one used for the Transit Navy Navigation Satellite System. (See Offshore Technology Conference paper OTC 1789 by Thomas A. Stansell, 1973.)

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TABLES

Table 1

Site	Passes	Std. Deviation, sec	Std. Deviation of Mean, sec
Honolulu	79		
Lat		1.67	0.19
Long		1.59	0.18
Pago Pago	24		
Lat		1.56	0.32
Long		1.63	0.33
Suva (1)	29		
Lat		1.86	0.35
Long		2.53	0.47
Rabaul	6		
Lat		2.25	0.92
Long		2.43	0.99
Guam	13		
Lat		1.88	0.52
Long		1.32	0.37
Ponape	25		
Lat		1.93	0.39
Long		1.62	0.32
Palau	48		
Lat		1.16	0.17
Long		1.84	0.26
Suva (2)	153		
Lat		1.84	0.15
Long		1.76	0.14
Wellington	46		
Lat		0.73	0.11
Long		1.13	0.17



Table 2

Site	Elev., m	E Passes	W Passes	E vs W, sec	
Buga	968	3	3	Lat	1.014N
				Long	2.418E
Calí	1326	3	3	Lat	0.36S
				Long	63.78W
Popayan I	1743	2	2	Lat	0.66N
				Long	48.51W
Popayan II	?	1	2	Lat	3.60S
				Long	61.98W
Laguna da Cocha	3133*	5	5	Lat	2.73S
				Long	155.27W
Tumaco	5	3	3	Lat	3.902N
				Long	0.36W
Buena Ventura	5*	3	3	Lat	2.142S
				Long	0.462W
Bogota	2650	1	4	Lat	7.72N
				Long	278.01W

Table taken from Woollard and Thompson, p. 38.

\* These elevations are in error in Woollard and Thompson.

TABLE 3

TEXACO MAP X, Y, COORDINATES CONVERTED TO GEOGRAPHICAL COORDINATES  
BY COMPUTER PROGRAM

C:L-1 1-00N 79-15W				
X 602440.0000	Y 423860.0000	N 0° 59.9799	W 79° 14.9989	
C:L-1 1-00N 78-30W				
602310.0000	507440.0000	0 59.9759	78 29.9890	
	65		.992	
C:L-2 1-00N 77-45W				
602220.0000	551340.0000	0 59.9831	77 44.9915	
C:L-3 1-00N 77-00W				
602160.0000	674950.0000	0 59.9960	76 59.9968	
C:L-4 1-00N 76-15W				
602110.0000	758490.0000	1 0.0041	76 15.0096	
C:J-4 4-00N 77-00W				
934320.0000	675690.0000	3 59.9930	76 59.9961	
C:J-4 3-00N 77-00W				
823630.0000	675320.0000	3 0.0110	77 0.0098	

TABLE 4

## COMPARISON OF REFERENCE POINT POSITIONS ON IGAC AND TEXACO MAPS

BUENA VENTURA-VALLE		260-III-C J-4	
925520.0000	1011500.0000	3 55.5307	76 58.6424
924580.0000	678060.0000	3 54.9363	76 58.6997
		0 0.5944	0 -0.0573
		260-III-C J-4	
924780.0000	1012330.0000	3 55.1292	76 58.1941
924200.0000	678930.0000	3 54.5153	76 58.2289
		0 0.6139	0 -0.0348
		260-III-C J-4	
926230.0000	1013310.0000	3 55.9158	76 57.6646
925620.0000	679910.0000	3 55.2866	76 57.7028
		0 0.6293	0 -0.0382
		260-III-C J-4	
920630.0000	1012470.0000	3 52.8774	76 58.1183
921020.0000	677620.0000	3 52.7896	76 58.9298
		0 0.0878	0 -0.8110
		260-III-C J-4	
921520.0000	1013390.0000	3 53.3602	76 57.6217
921780.0000	678680.0000	3 53.2034	76 58.3593
		0 0.1568	0 -0.7376
		279-I-A J-4	
919780.0000	1013690.0000	3 52.4161	76 57.4598
920180.0000	679120.0000	3 52.3372	76 58.1189
		0 0.0789	0 -0.6592
PALMIRA-VALLE		280-III-D J-4	
889540.0000	1089570.0000	3 35.9872	76 16.4865
889760.0000	756180.0000	3 35.9689	76 16.4921
		0 0.0182	0 -0.0056
		290-III-D J-4	
887860.0000	1085480.0000	3 35.0776	76 18.6957
888080.0000	752140.0000	3 35.0528	76 18.6699
		0 0.0249	0 0.0258
		280-III-D J-4	
883470.0000	1086190.0000	3 32.9668	76 18.3141
884190.0000	752780.0000	3 32.9445	76 18.3195
		0 0.0223	0 -0.0054
		280-III-D J-4	
880570.0000	1085830.0000	3 31.1223	76 18.5100
880760.0000	752480.0000	3 31.0844	76 18.4770
		0 0.0379	0 0.0331

(cont.)



TABLE 4 (cont.)

MEACADFRS-CAPLA	386-IV-D L-3			
686600.0000	991900.0000	1	45.3500	77 9.2227
686810.0000	657970.0000	1	45.3115	77 9.1956
		0	0.0385	0 0.0273
	386-IV-D L-3			
686000.0000	991700.0000	1	45.5671	77 9.2874
686200.0000	657870.0000	1	45.5228	77 9.2496
		0	0.0443	0 0.0379
	386-IV-B L-3			
691100.0000	990270.0000	1	48.3343	77 10.1018
691020.0000	656440.0000	1	48.1330	77 10.0239
		0	0.2013	0 0.0770
	386-IV-B L-3			
690650.0000	990660.0000	1	48.0902	77 9.9915
690910.0000	656490.0000	1	48.0737	77 9.8892
		0	0.0165	0 0.0023
	386-IV-B L-3			
693280.0000	989610.0000	1	49.5172	77 10.4577
693560.0000	655610.0000	1	49.5035	77 10.4731
		0	0.0087	0 -0.0154
TUQUERES-NARIND	428-IV-C L-2			
609920.0000	923190.0000	1	4.2805	77 46.2596
610150.0000	589080.0000	1	4.2756	77 46.2122
		0	0.0049	0 0.0472
	428-IV-C L-2			
609520.0000	922720.0000	1	4.0634	77 46.5127
609780.0000	588600.0000	1	4.0750	77 46.4702
		0	-0.0116	0 0.0425
	428-IV-C L-2			
603400.0000	924000.0000	1	0.7430	77 45.8220
603700.0000	589840.0000	1	0.7835	77 45.7993
		0	-0.0405	0 0.0227
	428-IV-C L-2			
608110.0000	924310.0000	1	3.2986	77 45.6554
608330.0000	590190.0000	1	3.2909	77 45.6139
		0	0.0077	0 0.0415
	428-IV-D L-3			
603310.0000	931690.0000	1	0.6950	77 41.6769
603620.0000	597530.0000	1	0.7449	77 41.6624
		0	-0.0499	0 0.0145
	428-IV-D L-3			
603540.0000	934830.0000	1	1.0371	77 39.9845
604180.0000	600670.0000	1	1.0501	77 39.9735
		0	-0.0129	0 0.0110

(cont.)

TABLE 4 (cont.)

CIYACANQUEM-NAMIMO		429-III-B L-3		
611990.0000	459580.0000	1	5.4071	77 26.6439
617230.0000	625420.0000	1	5.4250	77 26.6813
		0	-0.0179	0 -0.0174
		428-III-B L-3		
617130.0000	957970.0000	1	8.1766	77 22.1215
617320.0000	633400.0000	1	8.1874	77 22.1003
		0	0.0092	0 0.0207
CIPASTO-NARIND		429-III-C L-3		
625250.0000	980810.0000	1	12.6031	77 15.7001
625470.0000	646690.0000	1	12.6106	77 15.2221
		0	-0.0075	0 -0.0220
		429-III-C L-3		
624950.0000	983400.0000	1	12.4404	77 13.8039
625110.0000	649320.0000	1	12.4173	77 13.8062
		0	0.0232	0 -0.0924
		429-III-C L-3		
628130.0000	977190.0000	1	14.1657	77 17.1517
628360.0000	643080.0000	1	14.1740	77 17.1670
		0	-0.0083	0 -0.0153
		429-III-C L-3		
624420.0000	982470.0000	1	12.1420	77 14.3052
624570.0000	648370.0000	1	12.1241	77 14.3173
		0	0.0179	0 -0.0121
C:ME COA-PUTUMAYOX		430-IV-C L-4		
602600.0000	1040430.0000	1	0.3120	76 43.0614
602610.0000	706320.0000	1	0.2544	76 43.1071
		0	0.0577	0 -0.0457
		430-IV-D L-4		
603330.0000	1048000.0000	1	0.7076	76 38.9808
603340.0000	713900.0000	1	0.6533	76 39.0256
		0	0.0543	0 -0.0448
		430-IV-D L-4		
607970.0000	1049260.0000	1	3.2252	76 38.3013
607970.0000	715090.0000	1	3.1636	76 38.3868
		0	0.0616	0 -0.0855
		430-IV-D L-4		
606850.0000	1047630.0000	1	2.6176	76 39.1900
606800.0000	713500.0000	1	2.5287	76 39.2475
		0	0.0889	0 -0.0625
CIPTO-ASIS-PUTUMAYO		467-I-C M-4		
548980.0000	1063660.0000	0	31.2167	76 30.5435
545000.0000	729800.0000	0	31.2009	76 30.4463
		0	0.0158	0 0.0972
		467-I-C M-4		
546810.0000	1063970.0000	0	30.0302	76 30.3765
546830.0000	730110.0000	0	30.0246	76 30.2790
		0	0.0147	0 0.0976

TABLE 5

## Summary of Results of Table 4 by Area

Area	IGAC North of Texaco		IGAC West of Texaco		Number of Points
	Minutes of Arc	$\sigma$	Minutes of Arc	$\sigma$	
Buenaventura-Valle North of 922	+ 0.613	0.015	- 0.043	0.010	3
South of 922	+ 0.108	0.035	- 0.736	0.062	3
Palmira-Valle	+ 0.026	0.007	+ 0.020	0.018	4
Mercaderes-Cauca	+ 0.027	0.015	+ 0.013	0.021	4
Tuqueres-Nariño	- 0.017	0.021	+ 0.030	0.014	6
Yacanquer-Nariño	- 0.004	0.014	+ 0.002	0.019	2
Pasto-Nariño (east of)	+ 0.006	0.014	- 0.013	0.007	4
Mocoa-Putamayo (west of)	+ 0.066	0.014	- 0.060	0.017	4
PTO Asis-Putamayo	+ 0.015	0.001	+ 0.097	0.000	2
Average for 32 points		0.018		0.016	32
Average for 29 points excluding Buenaventura- South of 922		0.015		0.015	29
(No good reference points were available in the Buenaventura-South section)					
Coordinate corners (see text)					
From Texaco map	+ 0.0082	0.0130	+ 0.0008	0.0076	7



Table 6

Error in Longitude Position Away from True  
Position per Meter Error in Antenna Height with  
Increasing Satellite Elevation Angle,  
Based on Stansell (1970)

Error  $f(K)$ : in nautical miles per meter error in  
antenna height

Elevation  $K$ : degrees

$K$ (degrees)	$f(K) 10^{-4}$ n. miles
10	2.81
20	3.62
30	4.62
40	6.05
50	8.24
60	11.63
70	17.55

TABLE 7

Satellite Navigation Results for Each Observation Site  
(from Woollard and Thompson)

Time	Sat.	Dir.	Elev., K	Lat.	Long.	Initialized Height*	
						Geoid g	Antenna H
Buga, Colombia; Feb. 12, 1973; elev. 968 m						8	960
0700	41	S	46E	3°53.654'	76°18.137'		
0850	41	S	16W	53.682'	18.200'		
0946	53	S	38E	53.655'	18.158'		
1034	64	N	24E	53.621'	18.175'		
1136	53	S	18W	53.679'	18.198'		
1218	64	N	37W	53.648'	18.193'		
1418	62	N	64W	53.632'	18.197'		
Cali, Colombia; Feb. 14, 1973; elev. 1326 m						8	1326
0252	62	S	29W	3°25.725'	76°34.018'		
0538	63	S	16E	25.721'	34.768'		
0700	41	S	49E	25.783'	35.224'		
0726	63	S	44W	25.700'	33.798'		
1042	64	N	35E	25.673'	35.027'		
1228	64	N	26W	25.770'	34.014'		
Popayan (1), Colombia; Feb. 14-15, 1973; elev. 1743 m						8	1743
2130	53	N	29E	2°26.585'	76°37.043'		
2215	64	S	15E	26.695'	36.863'		
2316	53	N	25W	26.647'	36.115'		
0004	64	S	37W	26.611'	35.974'		
Laguna de la Cocha, Colombia; Feb. 18, 19, and 20, 1973; elev. 3133m						15	2808
1918	63	N	33W	1°08.172'	77°07.748'		
2012	41	N	22W	08.189'	08.025'		
2134	53	N	48E	08.004'	10.440'		
2238	64	S	37E	08.242'	09.897'		
2322	53	N	14W	08.145'	08.143'		
0026	64	S	16W	08.145'	08.170'		
0216	62	S	40W	08.047'	07.515'		
1822	41	S	35E	08.050'	09.905'		
1928	63	S	22W	08.197'	08.003'		
2008	41	S	20W	08.175'	08.058'		
2138	53	S	64E	07.92'	11.659'		
2248	64	N	56E	08.312'	10.797'		(c

(cont.)

TABLE 7 (cont.)

Time	Sat.	Dir.	Elev., K	Lat.	Long.	Initialized Height*	
						Geoid g	Antenna H
Tumaco, Colombia; Feb. 22, 1973; elev. 5 m						15	5
1818	41	N	33E	1°49.139'	78°43.846'		
1940	63	N	17W	49.084'	43.840'		
2142	53	N	74E	49.136'	43.854'		
2258	64	S	74E	49.177	43.838'		
Buenaventura, Colombia; Feb. 27, 1973; elev. 5 m						15	10
2100	53	N	52E	3°53.522'	77°03.939'		
2226	64	S	69E	53.535'	03.903'		
2248	53	N	13W	53.610'	03.899'		
0014	64	S	7W	53.545'	03.904'		
0200	62	S	26W	53.528'	03.903'		
0454	63	S	23E	53.519'	03.887'		
Bogota, Colombia; Mar. 2-4, 1973; Elev. 2650 m						10	2650
1258	62	N	56W	4°37.847'	76°0.969'		
0512	63	S	66E	37.732'	6.75'		
0628	41	S	59W	37.401'	1.530'		
0700	63	S	9W	37.537'	3.214'		
1358	62	N	12W	37.658'	2.753'		

\* Remarks: The lat. and long. columns have been taken directly from Thompson's original Satellite Navigation Log.



TABLE 8

## COLOMBIA SITE POSITION IGAC MAP

BUGA HOTEL GUADALAHARA 261-III-D-4				
ELEV: 973 METERS	X	Y	N	W
922430.0000		1086062.0000	3 53.8331	76 18.3650
POPAYANI HOTEL MONASTERIO 1:2000 POPAYAN 5				
ELEV: 1731.5 METERS				
762004.0000		1052120.0000	2 26.8026	76 36.7389
LAGUNA LACÓCHA HOTEL 429-IV-B				
ELEV: 2808 METERS				
617230.0000		992390.0000	1 8.2516	77 8.9574
TUMACO 4 M NE OF BENCHMARK 103TS-1,1951 383-IV-A				
ELEV: 3 METERS FROM SEA LEVEL				
692990.0000		816090.0000	1 49.3142	78 44.0126

ICAC Map Coordinates and Satellite-Determined Coordinates for Colombia

HEIGHT									
Map, m	Error Estimate, m	SAT NAV Double Pass		Height Difference, m	SAT NAV Without Sat no. 64		Height Difference, m	Approximate Bouguer Anomalies, mgl	
		H + G + Corr m	$\sigma_M$		H + G + Corr m	$\sigma_M$			
Buga	973	$\pm 10$	1010 $\pm 16$	37	1021 $\pm 10$		48	+ 25	
Cali	--		*1341.5 $\pm 79$		1397			- -	
Popayan I	1731	$\pm 3$	*1726 $\pm 45$	-5	1694		-37	-135	
Laguna la Cocha	2808	$\pm 20$	*2819 $\pm 60$	11	2771 $\pm 24$		-37	-175	
Tumaco	3	$\pm 2$	†19.8 $\pm 2$	16.8	†19.8 $\pm 2$		16.8	0	
Buenaventura	7*	$\pm 1$	9 $\pm 26$	2	-9		-16	- 75	
*Measured from slide; above water		*Assuming digital jump ( see text )							
		†From single passes without no. 64							

TABLE 10A

R/V KANA KEOKI 1971 POSITIONAL DATA, SUVA, FIJI  
MOORED TO DOLPHINS AT N/E KING'S WHARF. ANTENNA HEIGHT 75 METERS.

DATE	GMT	SAT	ELEV	GEOM	LATITUDE	LONGITUDE	IT	CTS	CTSQ	DEVIATION FROM THE MEAN (IN SECONDS OF ARC)		ITERATE	ELEVATE	DEVIATION
										LATITUDE	LONGITUDE			
202	2340	63	26	S-E	18 7 45.425	178 25 33.24E	2	28	12	-1.4	1.0	0	0	0
203	312	42	77	S-E	18 7 45.365	178 25 26.82E	2	34	17	-1.4	-5.5	0	-77	0
203	4 6	64	37	S-E	18 7 45.005	178 25 31.68E	2	33	15	-1.8	-0.6	0	0	0
203	502	42	8	S-W	18 7 45.545	178 25 29.28E	4	8	3	-1.3	-3.0	0	-8	0
203	604	64	21	S-W	18 7 45.665	178 25 32.58E	2	27	13	-1.1	0.3	0	0	0
203	752	65	78	N-E	18 7 47.645	178 25 24.96E	2	32	0	0.8	-7.3	0	-78	0
203	820	54	7	S-E	18 7 48.505	178 25 26.22E	3	0	0	11.7	-6.1	0	-7	-7
203	9 0	62	13	N-E	18 7 48.305	178 25 32.88E	2	20	9	1.5	0.6	0	-13	0
203	940	65	7	N-W	18 7 42.845	178 25 29.40E	5	0	0	-4.0	-2.9	-5	-7	0
203	1002	54	77	S-W	18 7 46.985	178 25 45.66E	3	32	1	0.2	12.4	0	-77	-77
203	1104	63	53	N-W	18 7 48.185	178 25 32.88E	2	34	17	1.4	0.6	0	0	0
203	1434	42	34	N-E	18 7 47.525	178 25 31.02E	2	32	16	0.7	-1.3	0	0	0
203	1542	64	18	N-E	18 7 48.185	178 25 32.04E	2	26	12	1.4	-0.2	0	0	0
203	1622	42	23	N-W	18 7 47.105	178 25 32.76E	2	27	13	0.3	0.5	0	0	0
203	1726	64	44	N-W	18 7 47.585	178 25 33.24E	2	34	17	0.8	1.0	0	0	0
203	1936	65	64	S-E	18 7 46.385	178 25 29.28E	2	26	17	-0.4	-3.0	0	0	0
203	2108	63	10	S-E	18 7 45.425	178 25 34.86E	2	14	6	-1.4	2.6	0	-10	0
203	2124	54	42	N-E	18 7 48.665	178 25 33.00E	2	15	0	1.9	0.7	0	0	0
203	2252	63	69	S-W	18 7 44.825	178 25 34.44E	2	32	1	-2.0	2.2	0	0	0
203	2312	54	17	N-W	18 7 47.945	178 25 32.76E	2	25	11	0.2	0.5	0	0	0
204	220	42	25	S-E	18 7 45.185	178 25 33.00E	2	29	8	-1.6	0.7	0	0	0
204	330	64	13	S-E	18 7 46.865	178 25 33.66E	2	17	1	0.1	1.4	0	-13	0
204	406	42	32	S-W	18 7 44.525	178 25 32.46E	2	30	14	-2.3	0.2	0	0	0
204	514	64	57	S-W	18 7 45.785	178 25 33.36E	2	35	16	-1.0	1.1	0	0	0
204	704	65	28	N-E	18 7 46.585	178 25 30.90E	2	28	13	0.2	-1.4	0	0	0
204	850	65	25	N-W	18 7 46.385	178 25 32.88E	2	27	12	-0.4	0.6	0	0	0
204	912	54	34	S-E	18 7 44.645	178 25 32.58E	2	31	15	-2.2	0.3	0	0	0
204	1016	63	57	N-E	18 7 48.485	178 25 30.78E	2	32	13	1.7	-1.5	0	0	0
204	1100	54	23	S-W	18 7 45.965	178 25 31.38E	2	23	9	-0.8	-0.9	0	0	0
204	1204	63	13	N-W	18 7 45.005	178 25 32.10E	2	20	9	-1.8	-0.2	0	-13	0
204	1342	42	9	N-E	18 7 42.965	178 25 31.56E	3	12	5	-3.8	-0.7	0	-9	0
204	1526	42	72	N-W	18 7 48.005	178 25 37.26E	2	35	17	1.2	5.0	0	0	0
204	1638	64	72	N-E	18 7 47.405	178 25 28.62E	2	36	17	0.6	-3.7	0	0	0
204	18 6	64	10	N-W	18 7 47.285	178 25 31.38E	6	14	7	0.5	-0.9	-6	-10	0
204	1850	65	26	S-E	18 7 45.125	178 25 30.78E	2	33	15	-1.7	-1.5	0	0	0
204	2036	54	17	N-E	18 7 48.185	178 25 31.02E	2	20	9	1.4	-1.3	0	0	0
204	2204	63	45	S-E	18 7 45.005	178 25 31.02E	2	31	16	-1.8	-1.3	0	0	0
204	2224	54	48	N-W	18 7 48.905	178 25 33.42E	2	28	12	2.1	1.1	0	0	0
204	2350	63	17	S-W	18 7 44.825	178 25 32.22E	2	22	9	-2.0	-0.1	0	0	0
205	128	42	7	S-E	18 7 53.345	178 25 34.56E	2	0	0	6.5	2.3	0	-7	0
205	310	42	76	S-E	18 7 44.405	178 25 22.56E	2	31	1	-2.4	-9.7	0	-76	0
205	4 4	64	55	S-E	18 7 45.905	178 25 31.02E	2	34	10	-0.9	-1.3	0	0	0
205	500	42	7	S-W	18 7 51.185	178 25 31.02E	6	0	0	4.4	-1.3	-6	-7	0
205	614	64	14	S-W	18 7 44.945	178 25 31.68E	2	21	10	-1.0	-0.6	0	-14	0
205	802	65	69	N-E	18 7 46.925	178 25 37.74E	2	32	15	0.1	5.5	0	0	0
205	822	54	11	S-E	18 7 47.285	178 25 34.09E	2	16	7	0.5	1.8	0	-11	0
205	926	63	21	N-E	18 7 48.485	178 25 31.92E	2	26	12	1.7	-0.4	0	0	0
205	1006	54	65	S-W	18 7 45.565	178 25 33.36E	2	35	16	-0.8	1.1	0	0	0
205	1114	63	37	N-W	18 7 48.905	178 25 33.36E	2	33	16	1.2	1.1	0	0	0
205	1432	42	37	N-E	18 7 48.245	178 25 30.48E	2	33	16	1.4	-1.8	0	0	0
205	1550	64	28	N-E	18 7 48.865	178 25 31.56E	2	31	14	3.1	-0.7	0	0	0
205	1618	42	21	N-W	18 7 47.345	178 25 33.00E	2	26	12	0.5	0.7	0	0	0
205	1736	64	30	N-W	18 7 47.345	178 25 33.24E	2	31	15	0.5	1.0	0	0	0
205	1804	65	8	S-E	18 7 45.785	178 25 33.12E	6	10	4	-1.0	0.8	-6	-8	0
205	1948	65	71	S-W	18 7 54.725	178 27 29.76E	7	32	3	7.9	117.5	-7	0	-71
205	2116	63	16	S-E	18 7 46.085	178 25 33.24E	2	22	9	-0.7	1.0	0	0	0
205	2134	54	64	N-E	18 7 47.825	178 25 30.24E	2	25	9	1.0	-2.0	0	0	0
205	2300	63	48	S-W	18 7 46.265	178 25 33.36E	2	32	12	-0.5	1.1	0	0	0
205	2320	54	12	N-W	18 7 46.085	178 25 31.26E	2	17	8	-0.7	-1.0	0	-12	0
206	216	42	27	S-E	18 7 44.465	178 25 32.34E	2	30	14	-2.3	0.1	0	0	0
206	338	64	21	S-E	18 7 46.565	178 25 32.22E	2	24	4	-0.2	-0.1	0	0	0
206	402	42	29	S-W	18 7 44.465	178 25 33.24E	2	30	14	-2.3	1.0	0	0	0
206	524	64	39	S-W	18 7 46.625	178 25 34.02E	2	32	16	-0.2	1.7	0	0	0
206	714	65	42	N-E	18 7 46.625	178 25 30.12E	2	31	14	-0.2	-2.2	0	0	0
206	902	65	16	N-W	18 7 46.385	178 25 31.92E	2	22	10	-0.4	-0.4	0	0	0
206	918	54	45	S-E	18 7 46.385	178 25 32.76E	2	27	11	-0.4	-0.5	0	0	0
206	1024	63	74	N-E	18 7 48.185	178 25 24.96E	2	31	0	1.4	-7.3	0	0	0
207	812	65	45	N-W	18 7 47.105	178 25 33.66E	2	30	11	0.3	1.4	0	0	0
207	936	63	30	N-E	18 7 47.28E	178 25 31.80E	2	27	11	0.5	-0.5	0	0	0
207	1012	54	50	S-W	18 7 47.10E	178 25 32.22E	2	30	14	0.3	-0.1	0	0	0
207	1122	63	26	N-W	18 7 46.50E	178 25 32.22E	2	29	14	-0.3	-0.1	0	0	0
207	1478	42	41	N-E	18 7 47.70E	178 25 30.78E	2	33	15	0.9	-1.5	0	0	0
207	1600	64	41	N-E	18 7 48.185	178 25 30.60E	2	32	4	1.4	-1.7	0	0	0
207	1620	42	19	N-W	18 7 48.425	178 25 32.10E	2	16	5	1.6	-0.2	0	0	0
207	1746	64	20	N-W	18 7 47.105	178 25 33.12E	2	27	12	0.3	0.8	0	0	0
207	1814	65	14	S-E	18 7 44.765	178 25 32.22E	2	24	11	-2.0	-0.1	0	-14	0
207	1958	65	64	S-W	18 7 46.725	178 25 36.18E	2	38	17	-0.5	3.9	0	0	0

TABLE II A

R/V KANA KEOKI 1971 POSITIONAL DATA, SUVA, FIJI  
 MOORED TO DOLPHINS AT N/E KING'S WHARF. ANTENNA HEIGHT 54 METERS.

DATE	GMT	SAT	ELEV	GEOM	LATITUDE	LONGITUDE	IT	CTS	CTSQ	DEVIATION FROM THE MEAN (IN SECONDS OF ARC)		ITERATE	ELEVATE	DEVIATION
										LATITUDE	LONGITUDE			
207	2124	63	24	S-E	18 7 45.785	178 25 32.348	2	28	13	-1.0	0.1	0	0	0
207	2310	63	33	S-W	18 7 45.905	178 25 32.220	2	29	7	-0.8	0.0	0	0	0
208	212	42	30	S-E	18 7 44.285	178 25 32.880	2	28	12	-2.5	0.7	0	0	0
208	348	64	31	S-E	18 7 45.785	178 25 32.880	2	28	6	-1.0	0.7	0	0	0
208	534	64	26	S-W	18 7 45.245	178 25 31.920	2	29	13	-1.5	-0.3	0	0	0
208	724	65	64	N-E	18 7 47.345	178 25 29.400	2	32	15	0.6	-2.8	0	0	0
208	850	63	9	N-E	18 8 8.045	178 25 30.360	7	14	6	21.3	-1.8	-7	-9	-9
208	914	65	9	N-W	18 7 40.625	178 25 31.260	2	12	5	-6.1	-0.0	0	-9	0
208	1032	63	70	N-W	18 7 48.665	178 25 33.240	2	34	17	1.9	1.0	0	0	0
208	1108	54	12	S-W	18 7 46.685	178 25 30.600	2	18	6	-0.1	-1.6	0	-12	0
208	1326	42	12	N-E	18 7 48.605	178 25 32.880	2	19	8	1.9	0.7	0	-12	0
208	1512	64	15	N-E	18 7 46.145	178 25 31.800	2	22	10	-0.6	-0.4	0	0	0
208	1656	64	53	N-W	18 7 48.065	178 25 32.220	2	33	1	1.3	0.0	0	0	0
208	1910	65	54	S-E	18 7 45.605	178 25 31.020	2	33	1	-1.1	-1.2	0	0	0
208	2042	54	30	N-E	18 7 47.525	178 25 31.920	2	32	15	0.8	-0.3	0	0	0
208	2202	65	17	S-W	18 7 46.565	178 25 31.440	2	17	7	-0.2	-0.8	0	0	0
208	2220	63	12	S-E	18 7 49.025	178 26 4.200	17	13	0	2.3	32.0	-17	-12	-12
208	2234	54	28	N-W	18 7 47.225	178 25 32.100	2	20	6	0.5	-0.1	0	0	0
209	10	63	7	S-W	18 7 48.905	178 25 29.280	4	0	0	2.2	-2.9	0	-7	0
209	122	42	8	S-E	18 7 53.525	178 25 30.600	8	7	4	6.8	-1.6	-8	-8	0
209	300	64	10	S-E	18 7 45.425	178 25 34.200	6	0	0	-1.3	2.0	-6	-10	0
209	444	64	70	S-W	18 7 45.605	178 25 32.340	3	32	1	-1.1	0.1	0	0	0
209	626	65	22	N-E	18 7 47.525	178 25 31.680	2	26	12	0.8	-0.5	0	0	0
209	822	65	30	N-W	18 7 46.505	178 25 33.240	2	27	10	-0.2	1.0	0	0	0
209	944	63	44	N-E	18 7 48.185	178 25 32.100	2	34	16	1.4	-0.1	0	0	0
209	1014	54	38	S-W	18 7 46.805	178 25 32.220	2	33	15	0.1	0.0	0	0	0
209	1132	63	18	N-W	18 7 46.925	178 25 32.880	2	24	11	0.2	0.7	0	0	0
209	1424	42	45	N-E	18 7 47.405	178 25 31.560	2	33	16	0.7	-0.6	0	0	0
209	1608	63	60	N-E	18 7 48.485	178 25 30.780	2	31	1	1.7	-1.4	0	0	0
209	1756	63	13	N-W	18 7 45.665	178 25 32.700	2	19	9	-1.1	0.5	0	-12	0
209	1822	65	22	S-E	18 7 45.665	178 25 32.040	2	30	14	-1.1	-0.2	0	0	0
209	1954	54	9	N-E	18 7 44.465	178 25 33.420	6	12	5	-2.3	1.2	-6	-9	0
209	2010	65	45	S-W	18 7 46.085	178 25 33.120	2	34	16	-0.7	0.9	0	0	0
209	2132	63	34	S-E	18 7 45.185	178 25 32.760	2	32	15	-1.6	0.6	0	0	0
209	2318	63	23	S-W	18 7 45.425	178 25 33.000	2	26	7	-1.3	0.8	0	0	0
210	208	42	33	S-E	18 7 45.065	178 25 32.100	2	31	15	-1.7	-0.1	0	0	0
210	356	42	26	S-W	18 7 46.265	177 29 48.780	4	20	1	1790.5	***	0	0	-26
210	544	54	19	S-W	18 7 45.965	178 25 31.260	4	22	3	-0.8	-0.9	0	0	0
210	734	65	66	N-W	18 7 46.145	178 25 45.300	2	29	0	-0.6	13.1	0	0	-66
210	856	63	16	N-E	18 7 48.005	178 25 33.420	2	15	1	1.3	1.2	0	0	0
210	924	54	73	S-E	18 7 46.205	178 25 34.980	2	27	1	-0.5	2.8	0	0	0
210	1042	63	49	N-W	18 7 48.245	178 25 31.920	2	32	14	1.5	-0.3	0	0	0
210	1112	54	8	S-W	18 7 50.645	178 25 32.220	8	0	0	3.9	0.0	-8	-8	0
210	1330	42	14	N-E	18 7 45.245	178 25 33.900	2	21	10	-1.5	1.7	0	-14	0
210	1516	42	54	N-W	18 7 48.365	178 25 33.660	2	29	8	1.6	1.5	0	0	0



TABLE 11A (cont.)

R/V KANA KEOKI 1971 POSITIONAL DATA, SUVA, FIJI  
 MOORED TO DOLPHINS AT N/E KING'S WHARF. ANTENNA HEIGHT 54 METERS.

DATE	GMT	SAT	ELEV	GEOM	LATITUDE	LONGITUDE	IT	CTS	CTSQ	DEVIATION FROM THE MEAN (IN SECONDS OF ARC)		ITERATE	ELEVATE	DEVIATION
										LATITUDE	LONGITUDE			
210	1706	64	36	N-W	18 7 47.585	178 25 33.120	2	33	16	0.8	0.9	0	0	0
210	1920	65	73	S-E	18 7 45.425	178 25 28.745	3	35	0	-1.3	-3.5	0	0	0
210	2046	63	11	S-E	18 7 46.985	178 25 33.780	2	18	9	0.2	1.6	0	-11	0
210	2110	65	11	S-W	18 7 44.405	178 25 31.140	2	15	7	-2.3	-1.1	0	-11	0
211	116	42	5	S-E	18 7 46.505	178 25 35.885	6	11	4	-0.2	3.7	-6	-9	0
211	300	42	73	S-W	18 7 45.605	178 25 34.020	2	33	1	-1.1	1.8	0	0	0
211	454	63	48	S-W	18 7 45.905	178 25 32.220	2	35	17	-0.8	0.0	0	0	0
211	646	65	34	N-E	18 7 47.405	178 25 31.260	2	29	13	0.7	-0.9	0	0	0
211	832	54	28	S-E	18 7 45.005	178 25 32.040	2	19	2	-1.7	-0.2	0	0	0
211	952	63	64	N-E	18 7 48.905	178 25 31.800	2	29	12	2.2	-0.4	0	0	0
211	1018	54	29	S-W	18 7 46.265	178 25 31.920	2	28	10	-0.5	-0.3	0	0	0
211	1140	63	11	N-W	18 7 43.265	178 25 33.240	5	16	6	-3.5	1.0	-5	-11	0
211	1420	42	50	N-E	18 7 46.985	178 25 32.460	2	33	16	0.2	0.3	0	0	0
211	1608	42	15	N-W	18 7 46.205	178 25 31.800	2	23	11	-0.5	-0.4	0	0	0
211	1628	64	52	N-E	18 7 50.945	178 24 1.140	10	0	0	10.2	-91.1	-10	0	-52
211	1808	64	7	N-W	18 7 45.245	178 25 29.580	6	3	1	-1.5	-2.6	-6	-7	0
211	1832	65	32	S-E	18 7 44.645	178 25 32.100	2	34	16	-2.1	-0.1	0	0	0
211	1956	54	13	N-E	18 7 47.165	178 25 33.090	2	20	3	0.4	0.8	0	-13	0
211	2018	65	31	S-W	18 7 45.785	178 25 34.020	2	35	17	-1.0	1.8	0	0	0
211	2142	54	59	N-W	18 7 48.905	178 25 33.240	2	18	9	2.2	1.0	0	0	0
211	2328	54	17	N-W	22 51 38.105	167 29 27.540	5	0	0	*****	*****	-5	0	-17
212	206	42	37	S-E	18 7 45.245	178 25 32.460	6	0	0	-1.5	0.3	-6	0	0
212	352	42	21	S-W	18 7 44.765	178 25 32.100	2	27	13	-2.0	-0.1	0	0	0
212	412	63	66	S-E	18 7 46.985	178 25 32.880	2	19	2	0.2	0.7	0	0	0
212	554	63	11	S-W	18 7 46.925	178 25 31.380	2	17	8	0.2	-0.8	0	-11	0
212	904	63	24	N-E	18 7 48.185	178 25 32.340	2	29	14	1.4	0.1	0	0	0
212	930	54	71	S-W	18 7 45.185	178 25 24.125	2	23	1	-1.6	-8.1	0	0	0
212	1050	63	34	N-W	18 7 46.865	178 25 30.360	2	30	14	0.1	-1.8	0	0	0
212	1118	54	7	S-W	18 8 13.685	178 25 36.840	3	0	0	26.9	4.6	0	-7	-7
212	1328	42	16	N-E	18 7 54.365	178 25 31.920	2	23	10	7.6	-0.3	0	0	0
212	1512	42	49	N-W	18 7 48.125	178 25 32.880	2	33	16	1.4	0.7	0	0	0
212	1532	64	33	N-E	18 7 47.105	178 25 32.340	2	30	15	0.4	0.1	0	0	0
212	1716	64	25	N-W	18 7 46.205	178 25 32.580	2	28	11	-0.5	0.4	0	0	0
212	1746	65	11	S-E	18 7 44.645	178 25 33.240	2	19	8	-2.1	1.0	0	-11	0
212	1930	65	74	S-W	18 7 45.545	178 25 35.940	2	39	19	-1.2	3.7	0	0	0
212	2054	54	35	N-E	18 7 48.125	178 25 31.800	2	15	0	1.4	-0.4	0	0	0
212	2238	65	40	S-W	17 37 28.625	186 1 52.560	5	21	0	*****	*****	-5	0	-40
213	112	42	22	S-E	18 7 47.225	178 25 32.880	5	14	7	0.5	0.7	-5	0	0
213	256	42	67	S-W	18 7 45.845	178 25 33.420	2	34	16	-0.9	1.2	0	0	0
213	318	64	25	S-E	18 7 46.385	178 25 32.700	2	28	14	-0.4	0.5	0	0	0
213	504	64	32	S-W	18 7 46.265	178 25 32.220	2	32	15	-0.5	0.0	0	0	0
213	656	65	52	N-E	18 7 47.705	178 25 30.060	2	31	14	1.0	-2.1	0	0	0
213	818	63	7	N-E	18 7 52.265	178 25 35.940	9	0	0	5.5	3.7	-9	-7	0
213	826	54	37	S-E	18 7 45.185	178 25 32.700	2	33	16	-1.6	0.5	0	0	0
213	1002	63	73	N-E	18 7 48.965	178 25 34.740	15	0	0	2.2	2.5	15	0	0
213	1026	54	22	S-W	18 7 47.345	178 25 31.560	2	20	9	0.6	-0.6	0	0	0
213	1150	63	7	N-W	18 7 45.125	178 25 27.480	2	0	0	-1.6	-4.7	0	-7	0
213	1418	42	55	N-E	18 7 47.525	178 25 31.560	2	33	14	0.8	-0.6	0	0	0
213	1444	64	11	N-E	18 7 49.025	178 25 32.040	2	17	7	2.3	-0.2	0	-11	0
213	1606	42	14	N-W	18 7 45.905	178 25 32.100	2	20	9	-0.8	-0.1	0	-14	0
213	1628	64	65	N-W	18 7 48.245	178 25 33.360	2	36	17	1.5	1.2	0	0	0
213	1842	65	46	S-E	18 7 44.645	178 25 30.900	2	36	12	-2.1	-1.3	0	0	0
213	2000	54	18	N-E	18 7 48.665	178 25 32.100	2	25	11	1.9	-0.1	0	0	0
213	2028	65	21	S-W	18 7 46.205	178 25 34.080	2	28	12	-0.5	1.9	0	0	0
213	2144	54	45	N-W	18 7 48.485	178 25 32.340	2	27	11	1.7	0.1	0	0	0
213	2338	63	9	S-W	18 7 50.765	178 25 32.100	6	11	4	4.0	-0.1	-6	-9	0
214	202	42	40	S-E	18 7 45.125	178 25 32.700	2	33	16	-1.6	0.5	0	0	0
214	232	64	7	S-E	18 7 52.925	178 25 34.740	3	4	1	6.2	2.5	0	-7	0
214	350	42	19	S-W	18 7 46.085	178 25 30.900	2	26	12	-0.7	-1.3	0	0	0
214	414	64	68	S-W	18 7 46.745	178 25 35.280	2	33	1	-0.0	3.1	0	0	0
214	608	65	18	N-E	18 7 47.045	178 25 31.680	2	23	10	0.3	-0.5	0	0	0
214	748	54	12	S-E	18 7 42.605	178 25 35.940	2	19	9	-4.1	3.7	0	-12	0
214	912	63	34	N-E	18 7 47.105	178 25 33.120	2	30	11	0.4	0.9	0	0	0
214	932	54	62	S-W	18 7 45.005	178 25 31.140	2	30	8	-1.7	-1.1	0	0	0
214	1100	63	24	N-W	18 7 46.085	178 25 31.680	2	28	13	-0.7	-0.5	0	0	0
214	1324	42	17	N-E	18 7 47.525	178 25 32.580	2	24	2	0.8	0.4	0	0	0
214	1510	42	45	N-W	18 7 48.365	178 25 32.880	2	34	16	1.6	0.7	0	0	0
214	1540	64	49	N-E	18 7 47.165	178 25 31.800	2	35	16	0.4	-0.4	0	0	0
214	1726	64	16	N-W	18 7 45.665	178 25 31.920	2	24	11	-1.1	-0.3	0	0	0
214	1756	65	18	S-E	18 7 45.125	178 25 33.000	2	27	12	-1.6	0.8	0	0	0
214	1940	65	52	S-W	18 7 46.445	178 25 34.080	2	38	18	-0.3	1.9	0	0	0
214	2054	54	67	N-E	18 7 48.605	178 25 29.940	2	34	15	1.9	-2.3	0	0	0

TABLE IOB

			ARITHMETIC MEAN SOLUTION AT SUVA ANT 75 METERS		
NP	N	NSD	LATITUDE LONGITUDE	STANDARD DEVIATION (SECONDS)	STANDARD DEVIATION OF THE MEAN (SECONDS)
77	21	56	18 7 46.795 178 25 32.27E	1.3 1.9	0.2 0.3

TABLE IO C

			BY INDIVIDUAL SATELLITE - ARITHMETIC MEAN SOLUTION AT SUVA ANT 75 METERS		
SATELLITE NUMBER	NSC		LATITUDE LONGITUDE	STANDARD DEVIATION (SECONDS)	STANDARD DEVIATION OF THE MEAN (SECONDS)
42	11		18 7 46.63S 178 25 32.58E	1.6 1.8	0.5 0.5
54	10		18 7 47.06S 178 25 32.27E	1.4 1.1	0.4 0.3
63	13		18 7 46.73S 178 25 31.96E	1.4 2.3	0.4 0.6
64	13		18 7 47.01S 178 25 32.10E	1.3 1.4	0.4 0.4
65	9		18 7 46.47S 178 25 32.61E	0.6 2.8	0.2 0.9

TABLE IIB

			ARITHMETIC MEAN SOLUTION AT SUVA ANT 54 METERS		
NP	N	NSD	LATITUDE LONGITUDE	STANDARD DEVIATION (SECONDS)	STANDARD DEVIATION OF THE MEAN (SECONDS)
117	36	81	18 7 46.74S 178 25 32.20E	1.5 1.5	0.2 0.2

TABLE IIC

			BY INDIVIDUAL SATELLITE - ARITHMETIC MEAN SOLUTION AT SUVA ANT 54 METERS		
SATELLITE NUMBER	NSC		LATITUDE LONGITUDE	STANDARD DEVIATION (SECONDS)	STANDARD DEVIATION OF THE MEAN (SECONDS)
42	16		18 7 46.97S 178 25 32.46E	2.4 0.8	0.6 0.2
54	15		18 7 46.96S 178 25 31.61E	1.4 2.3	0.4 0.6
63	17		18 7 47.10S 178 25 32.30E	1.2 0.8	0.3 0.2
64	15		18 7 46.54S 178 25 32.51E	0.9 0.9	0.2 0.2
65	14		18 7 46.18S 178 25 32.10E	1.0 1.8	0.2 0.4

Suva, 1971. Longitude East of  $178^{\circ}25.5'$  (in min x  $10^4$ ) from Double Pass Method

Day	42	54	63	64	05	Daily Average LONG	$\sigma$ single obs.	$\sigma$ (mean)	Deviation from Average LONG (+=-), m
203	336	473	480	406	369	413	62	31	+ 3.87
204	378	290	300	306	321	347	61	25	- 7.75
205	573	181	417	546	253	432	175	71	+ 7.22
206	463				486	389	148	105	- 0.35
207	270		337	382	502	372	85	42	- 3.35
208		335		395	221	286	95	48	-18.49
209			437	484	402	421	41	20	+ 5.28
210	652		498		140	428	215	124	+ 6.52
211	331	509	498		511	436	96	48	+ 7.92
212	366	375	247	412	271	594	124	55	- 2.29
213	329	500	339	377	413	498	71	29	+ 2.11
214	447	261	386	314	564	394	118	59	+ 0.53
Average	389	439	400	430	372	365	274	429	
$\sigma$ (single obs)	+119	$\pm 108$	$\pm 90$	107	$\pm 37$	$\pm 89$	$\pm 107$	$\pm 165$	
Satellite average	410	389	409	369	377				
$\sigma$ (single obs)	$\pm 112$	$\pm 126$	$\pm 92$	$\pm 61$	$\pm 162$				
$\sigma_{\text{mean}}$	$\pm 31$	$\pm 40$	$\pm 26$	$\pm 17$	$\pm 45$				

Average, all satellites 391.0 (= 19.6 m)  
 $\sigma$ (single obs) 111.3 (= 2.47 m)  
 $\sigma$ (mean) 14.0

TABLE 13

Antenna Height: Satellites (42, 54, 63, 64)



## FIGURES

Fig. 1. Tangents of the satellite elevation angle as a function of the longitude fix, as given by the Magnavox 706 marine-type satellite navigator for Laguna la Cocha. Initialized antenna height was 2808 m; initialized geoid height was 15 meters.

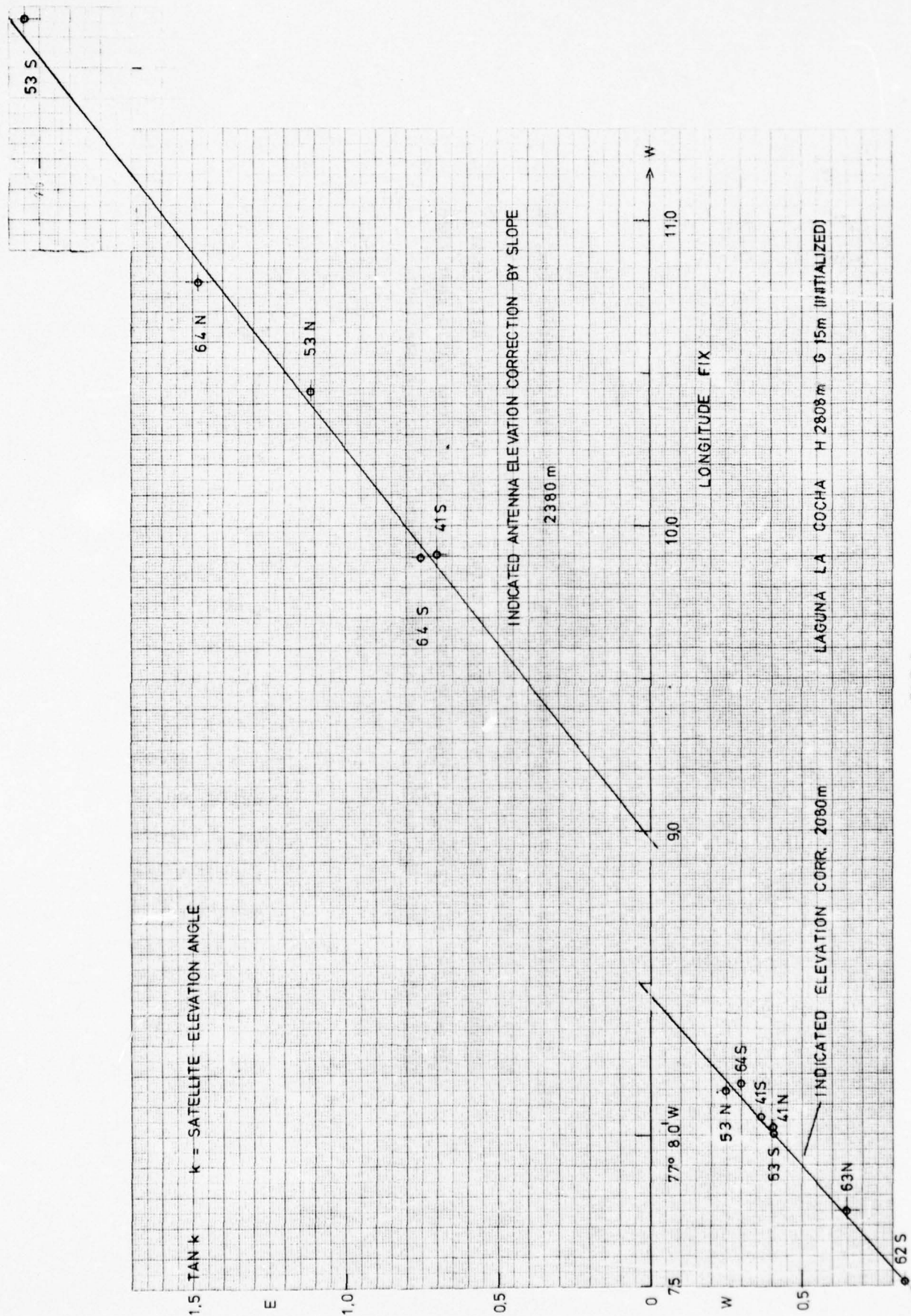


FIG.1

Fig. 2. Satellite elevation angle function as a function of the longitude fix, as given by the Magnavox 706 satellite navigator for Laguna la Cocha. Initialized antenna and geoid heights were 2808 and 15 meters. The longitude shift toward the west for east-passing satellites indicates that the antenna was higher than initialized. For the amount of the antenna elevation correction see text.



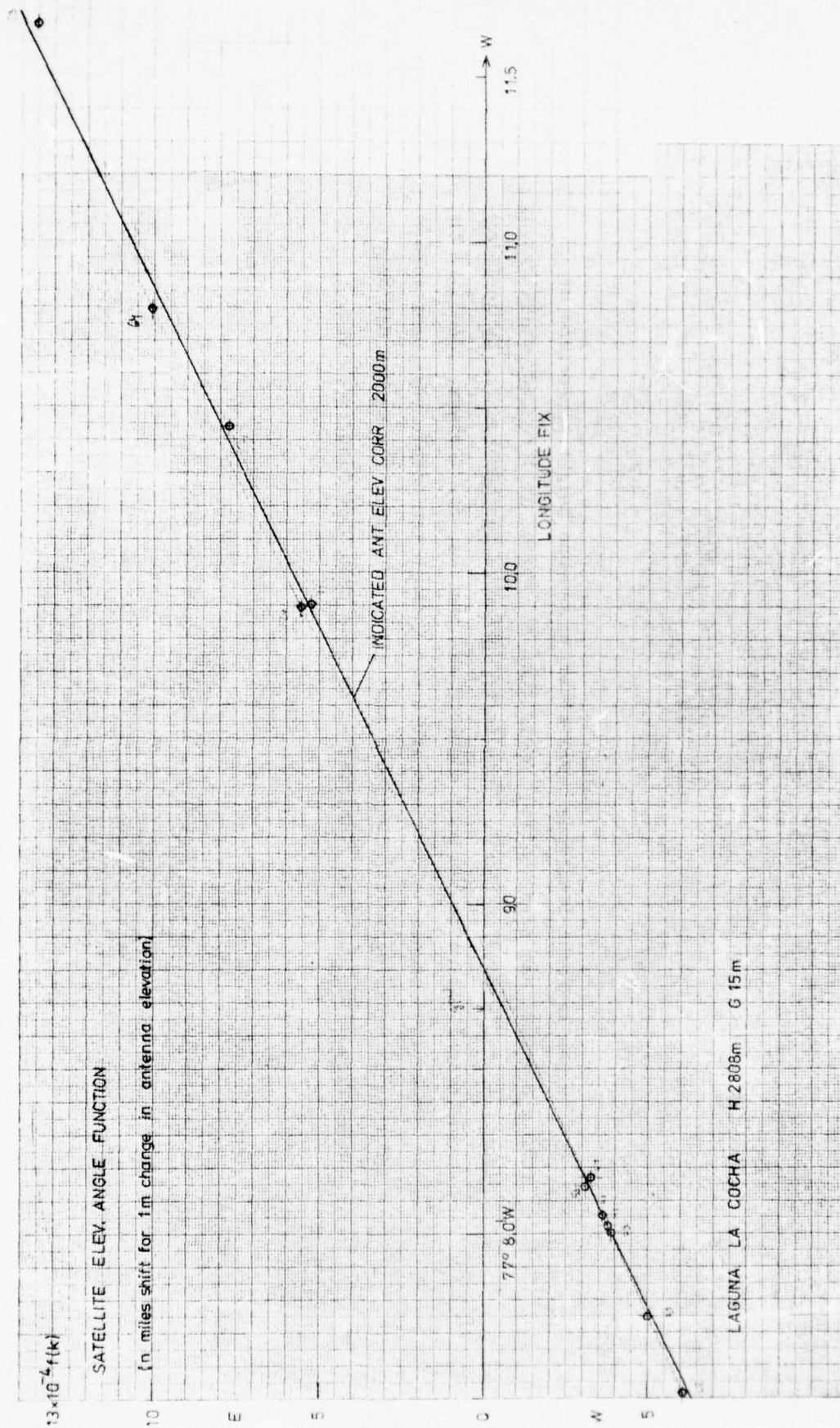


FIG.2

Fig. 3. Laguna la Cocha longitude fixes after an antenna height correction of 2110 m downward has been applied. Note that the spread in longitude still ranges from  $77^{\circ}8.643'W$  to  $77^{\circ}8.858'W$ . This large spread is further reduced by applying an additional correction to each double pass, that is, for the same satellite on two consecutive passes. The lines are drawn through the double pass positions, and the "0" crossing on the longitude axis gives the fix. The spread now only ranges from  $77^{\circ}8.7996'W$  to  $77^{\circ}8.8083'W$ , or less than 0.01 minutes of arc.

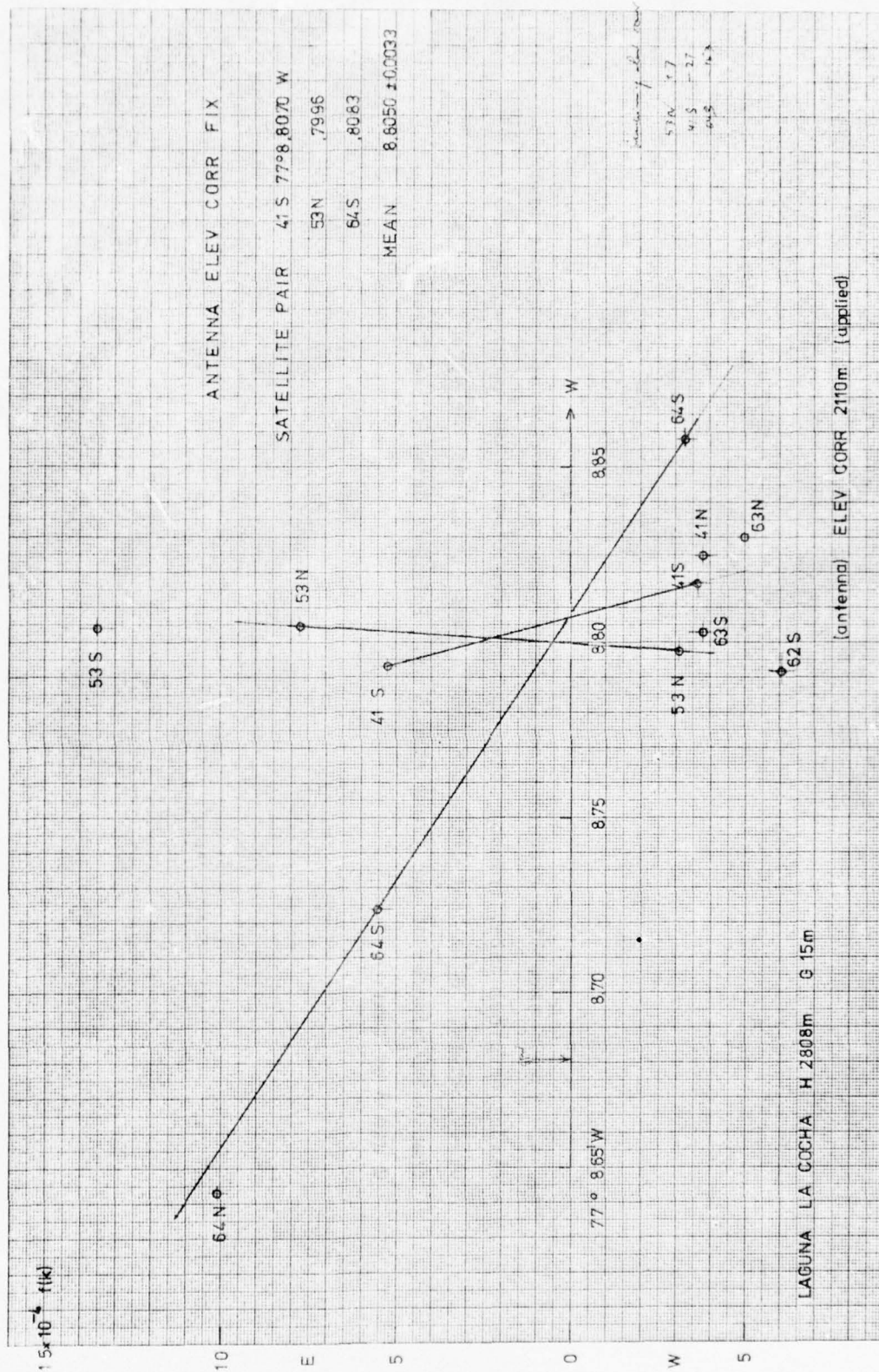


FIG.3

Fig. 4. Buga. Satellite elevation angle function as a function of longitude fix:  
(left) original data; (right) after upward correction of the antenna height  
by 44 meters. Note that after the double pass correction the spread is less  
than 0.003 minutes of arc (less than 5 m). For more details see Figures  
2 and 3 or text.



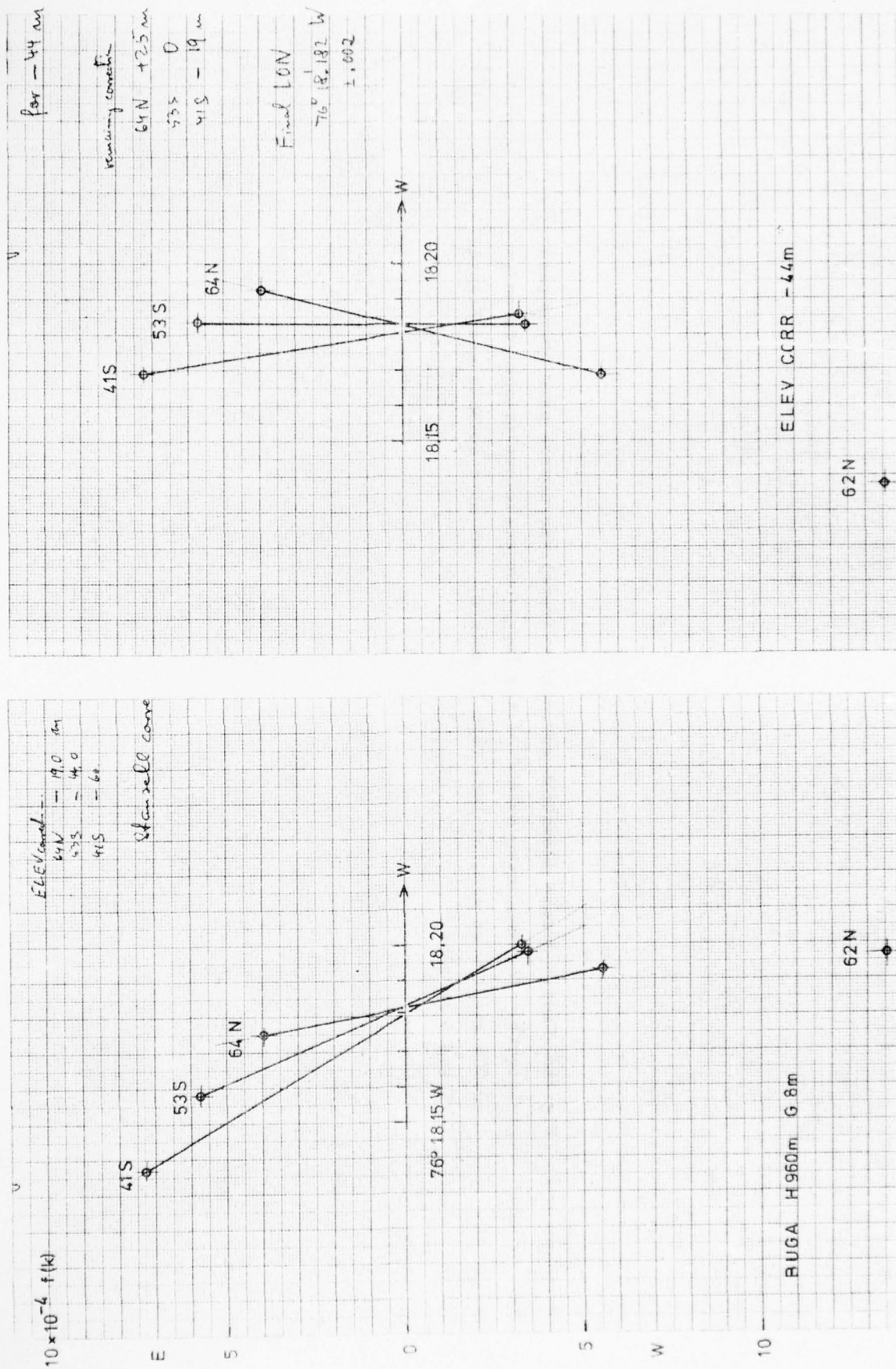
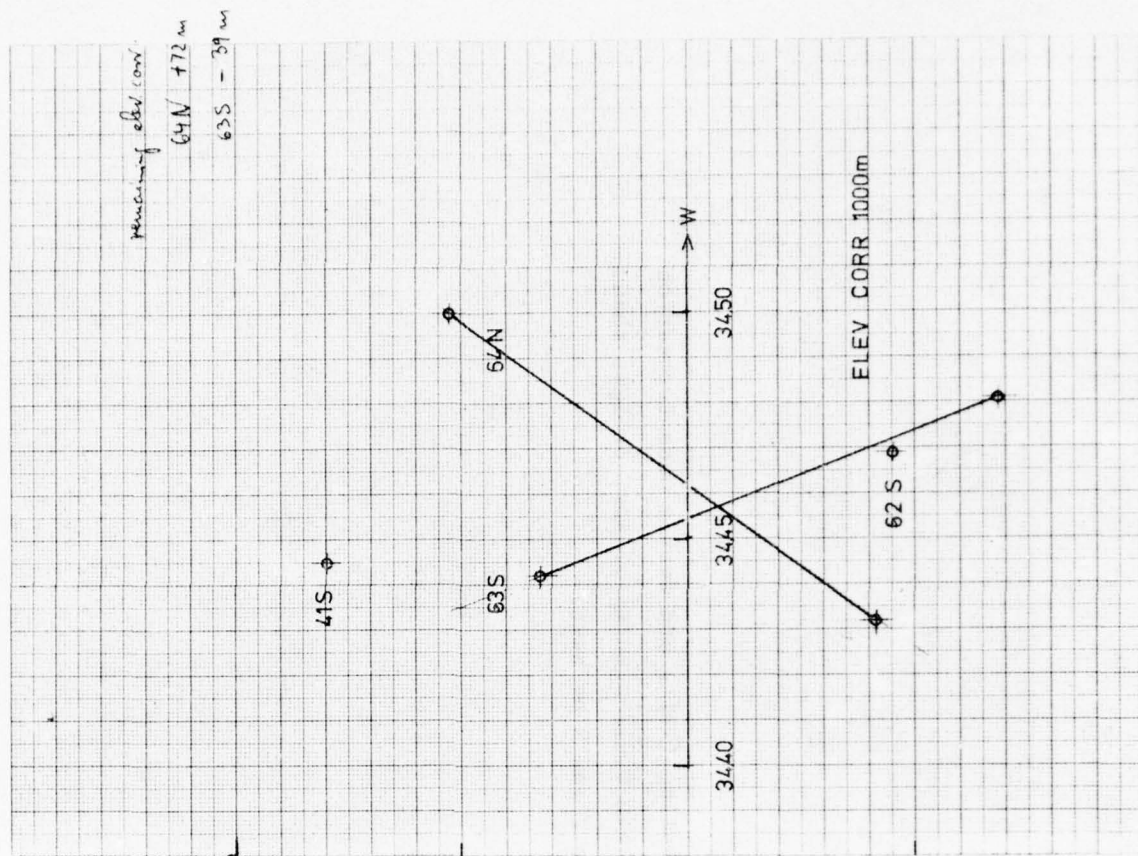
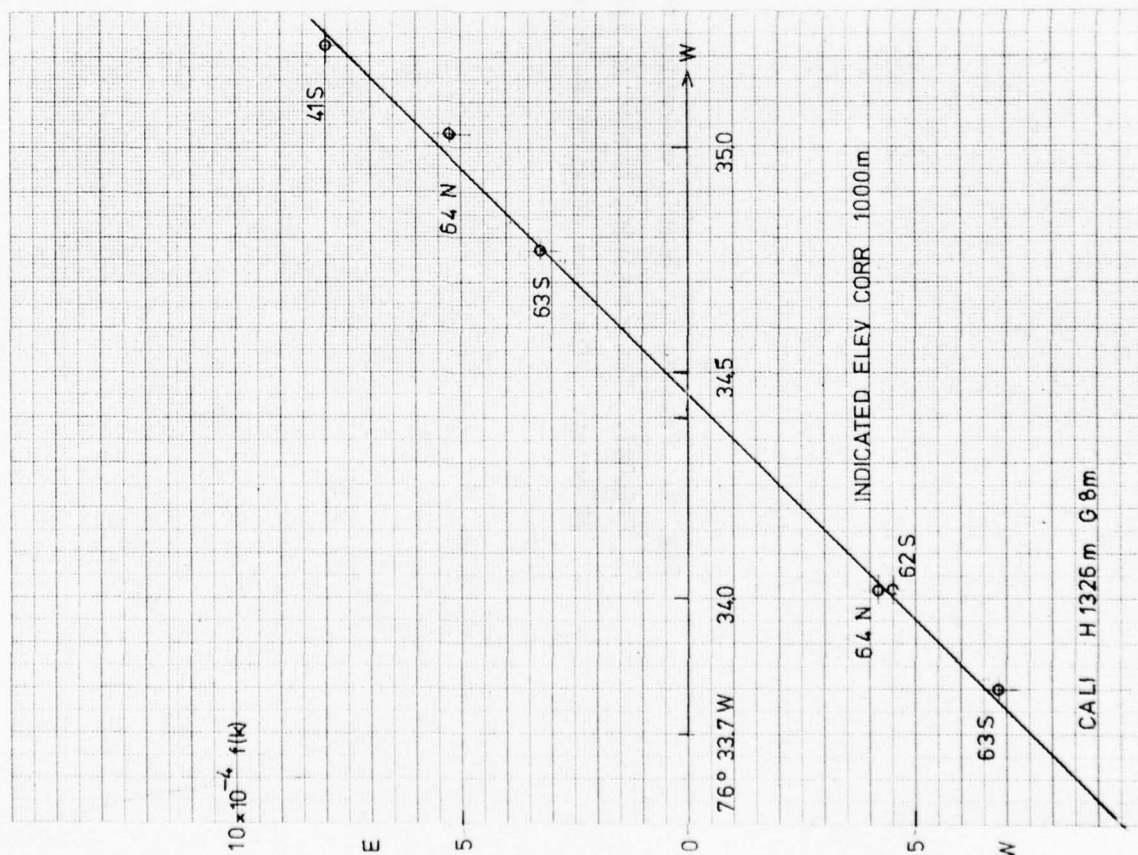


FIG.4

Fig. 5. Cali. Simple elevation correction (left) and double pass correction (right). The remaining spread of the longitude positions (right) is only 0.008 minutes of arc, or less than 15 meters. For more details see Figures 2 and 3 or text.



remaining elev corr.  
 64N +72 m  
 63S - 39 m

FIG. 5

Fig. 6. Popayan I. Simple elevation correction (left) and double pass correction (right). The remaining spread is less than 0.017 minutes of arc, or less than 31 meters. For more details see Figures 2 and 3 or text.



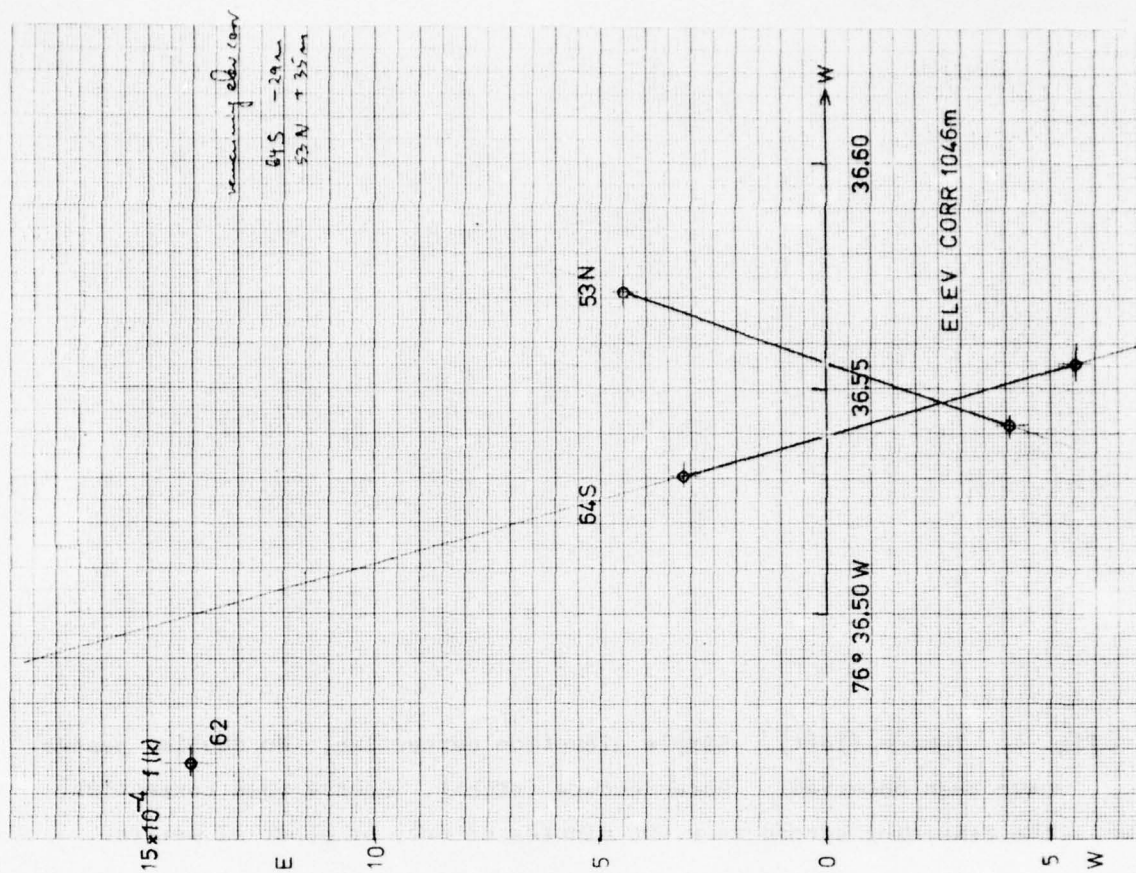
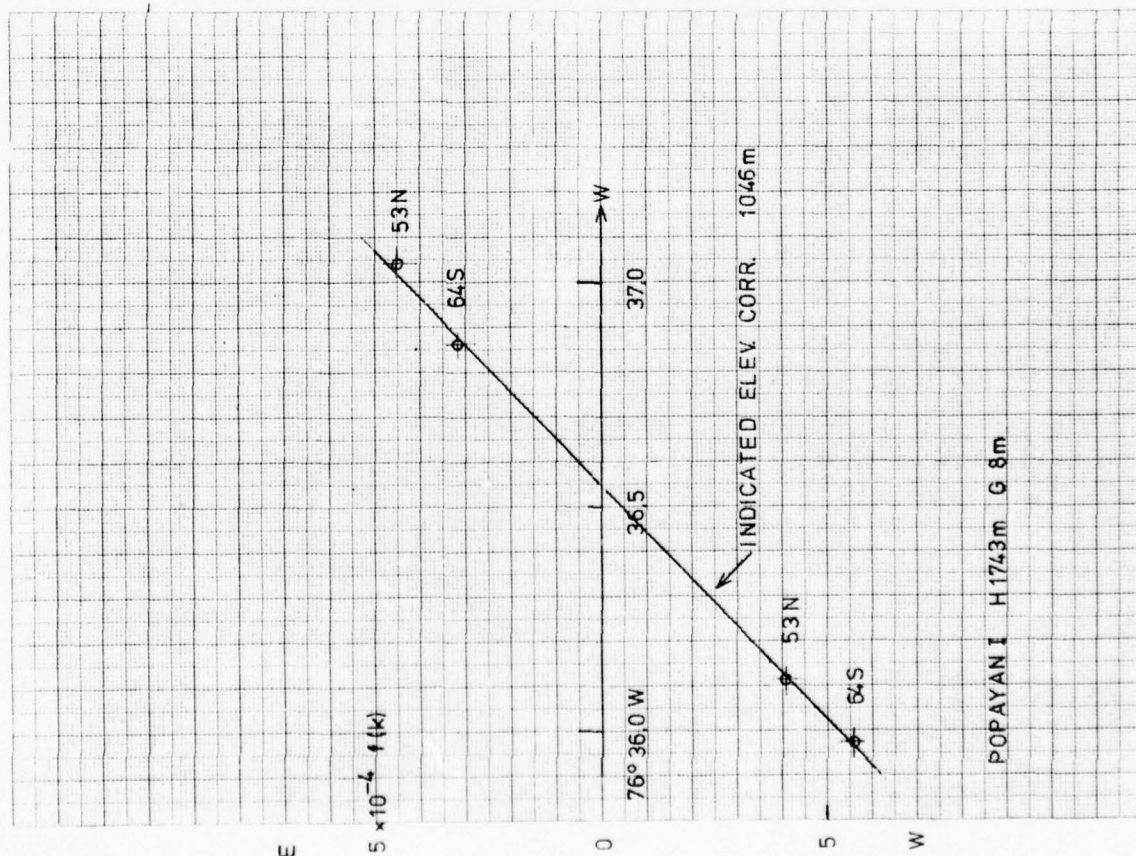


FIG. 6

Fig. 7. Tumaco (left). Simple elevation correction. No double passes have been observed. Buenaventura (right). Double pass correction. The remaining spread is 0.006 minutes of arc, or about 11 meters.





Fig. 8. Left: Double pass correction for the Suva harbor position on-board R/V KANA KEOKI for day 203 in 1971; the number below the satellite identification gives the chronological order in which double passes occurred. Note that 63S5 clearly is outside of the remaining five double pass longitude positions and therefore has been rejected. The spread of the data is only 25 meters. The standard deviation of the mean of the five double pass observations is 0.0031 minutes of arc, or 5.5 meters.

Right: Double pass positions for day 204.





Fig. 9. Left: Satellite elevation angle function (right side of line) as function of the satellite elevation angle (left side of line), as taken from Stansell (1970); see text.

Right: Double pass positions, Suva harbor, day 205, 1971.

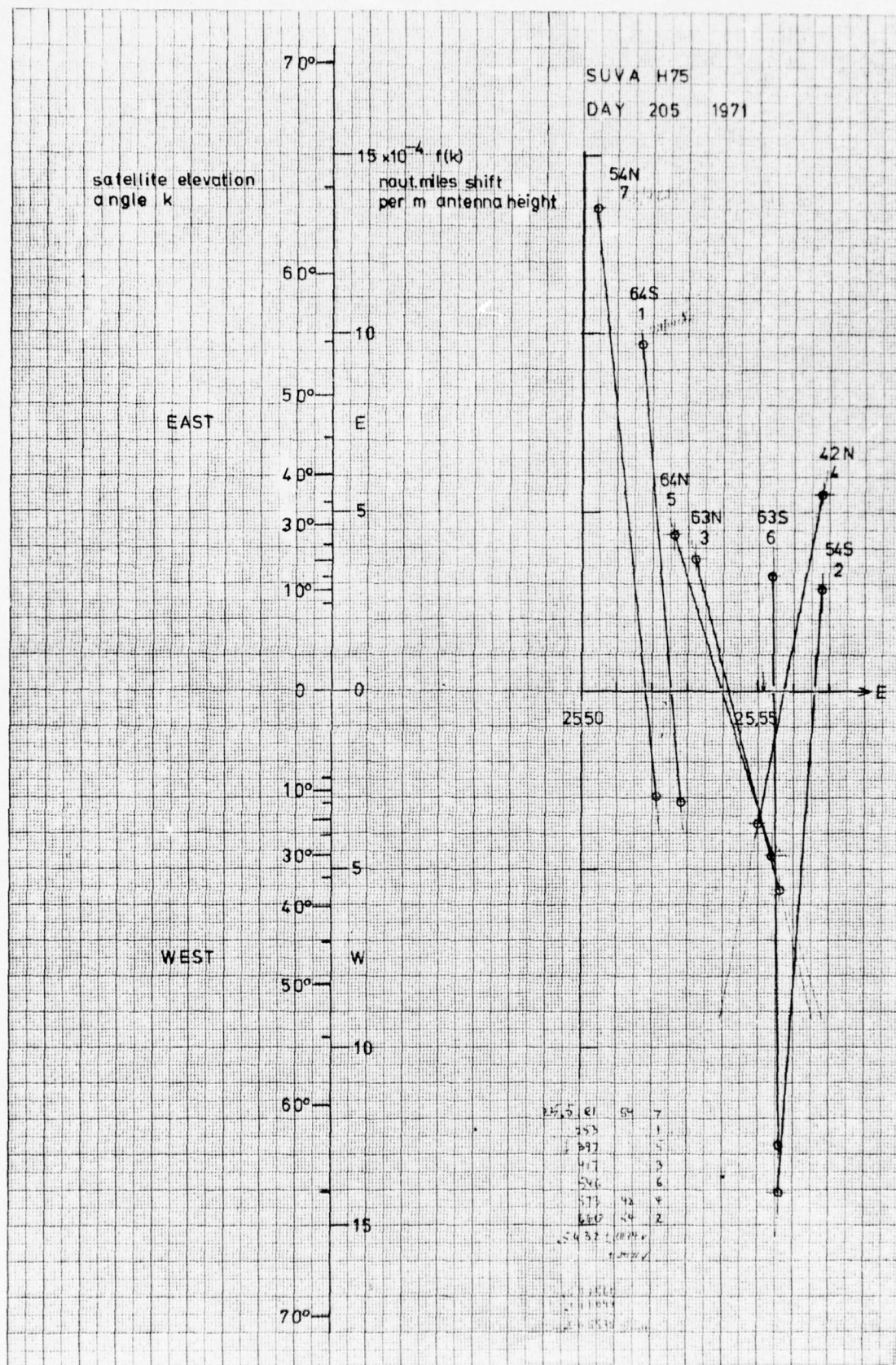


FIG 9



Fig. 10. Double pass positions for Suva harbor onboard R/V KANA KEOKI  
for days 206 and 207, 1971.



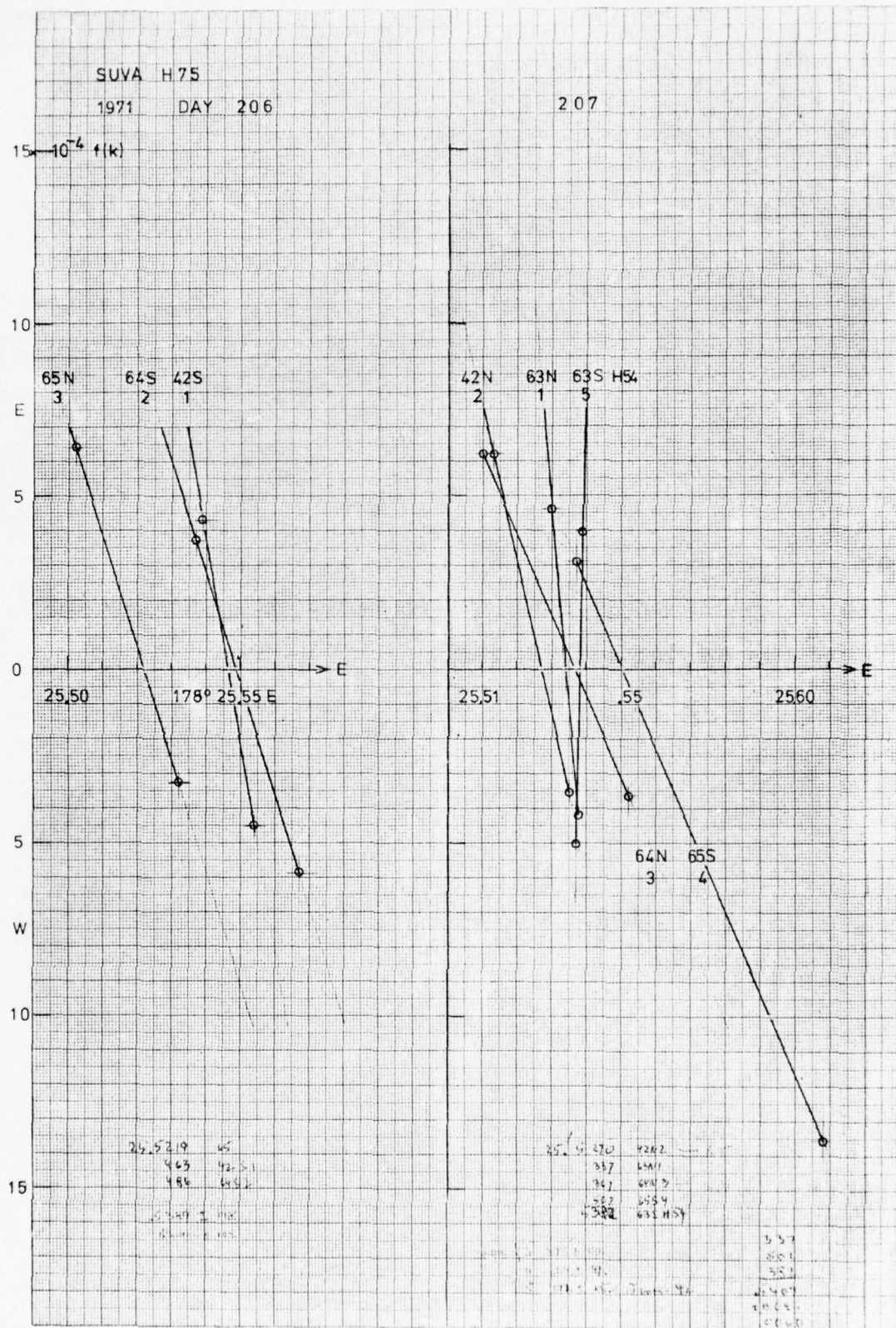


FIG.10

Fig. 11. Double pass positions for Suva harbor onboard R/V KANA KEOKI  
for days 208, 209, and 210, 1971.



Fig. 12. Double pass positions for Suva harbor onboard R/V KANA KEOKI  
for days 211 and 212, 1971.



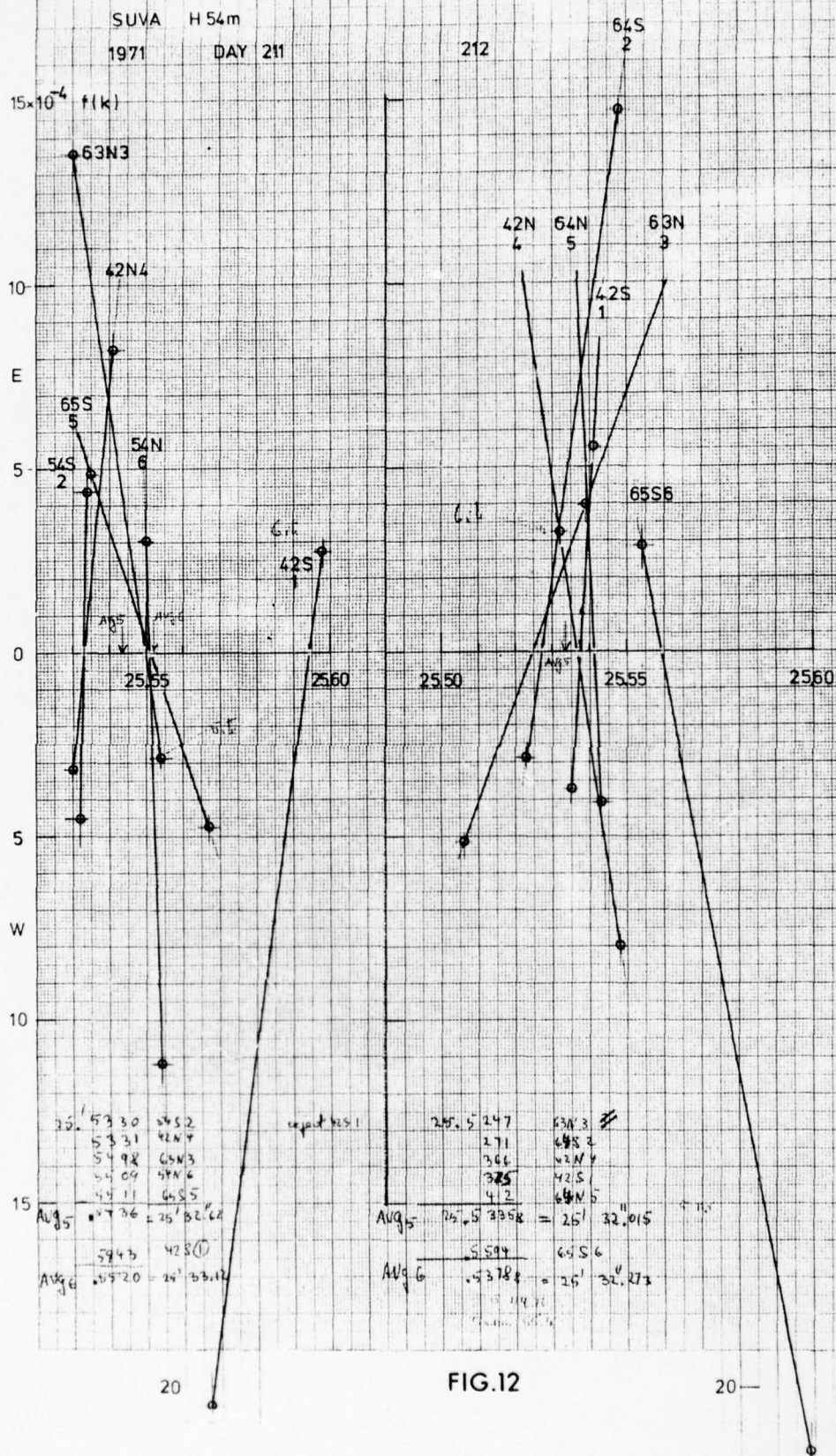


Fig. 13. Double pass positions for Suva harbor onboard R/V KANA KEOKI  
for days 213 and 214, 1971.





Fig. 14. Antenna height for satellite observations minus heights indicated by the IGAC maps as a function of Bouguer gravity anomaly for five Colombia sites (see text).



# ANTENNA - MAP HEIGHT

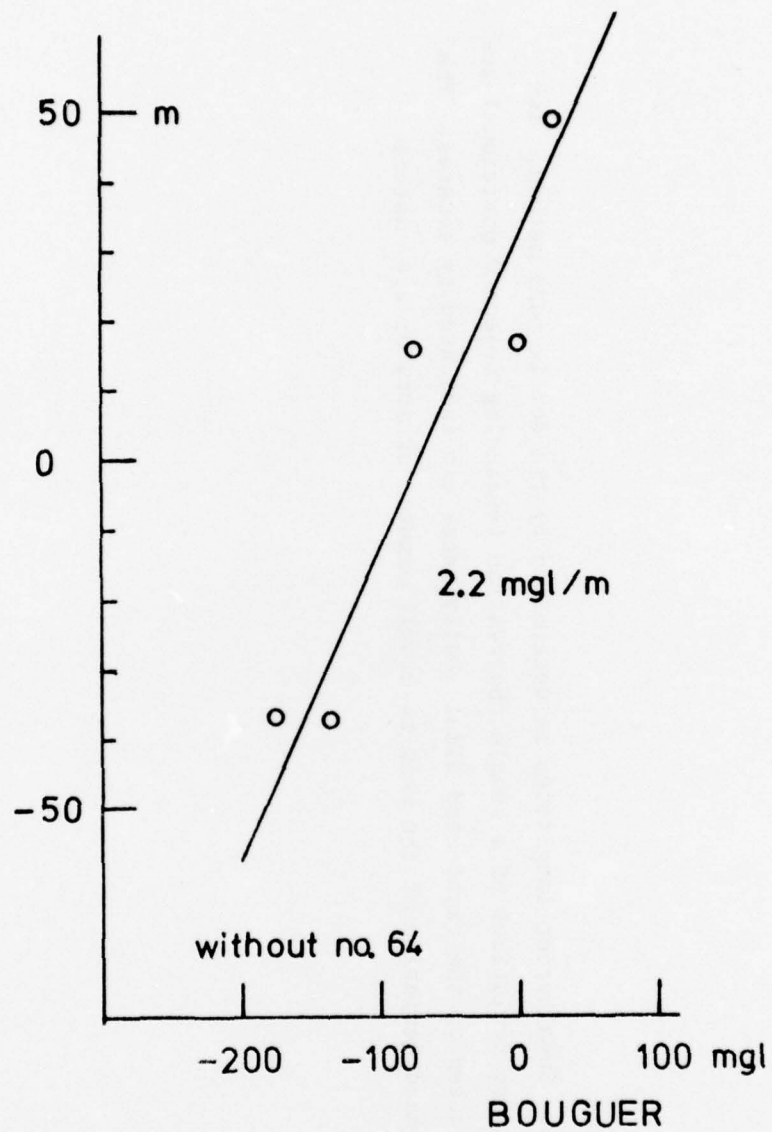


FIG.14

Fig. 15. Suva harbor longitudes as determined by the double pass method. Two standard deviations of a single observation (excluding rejected positions) are indicated on the right-hand side. Daily means are indicated by squares. The standard deviation of the mean is 0.0014 minutes of arc, or 2.47 meters.

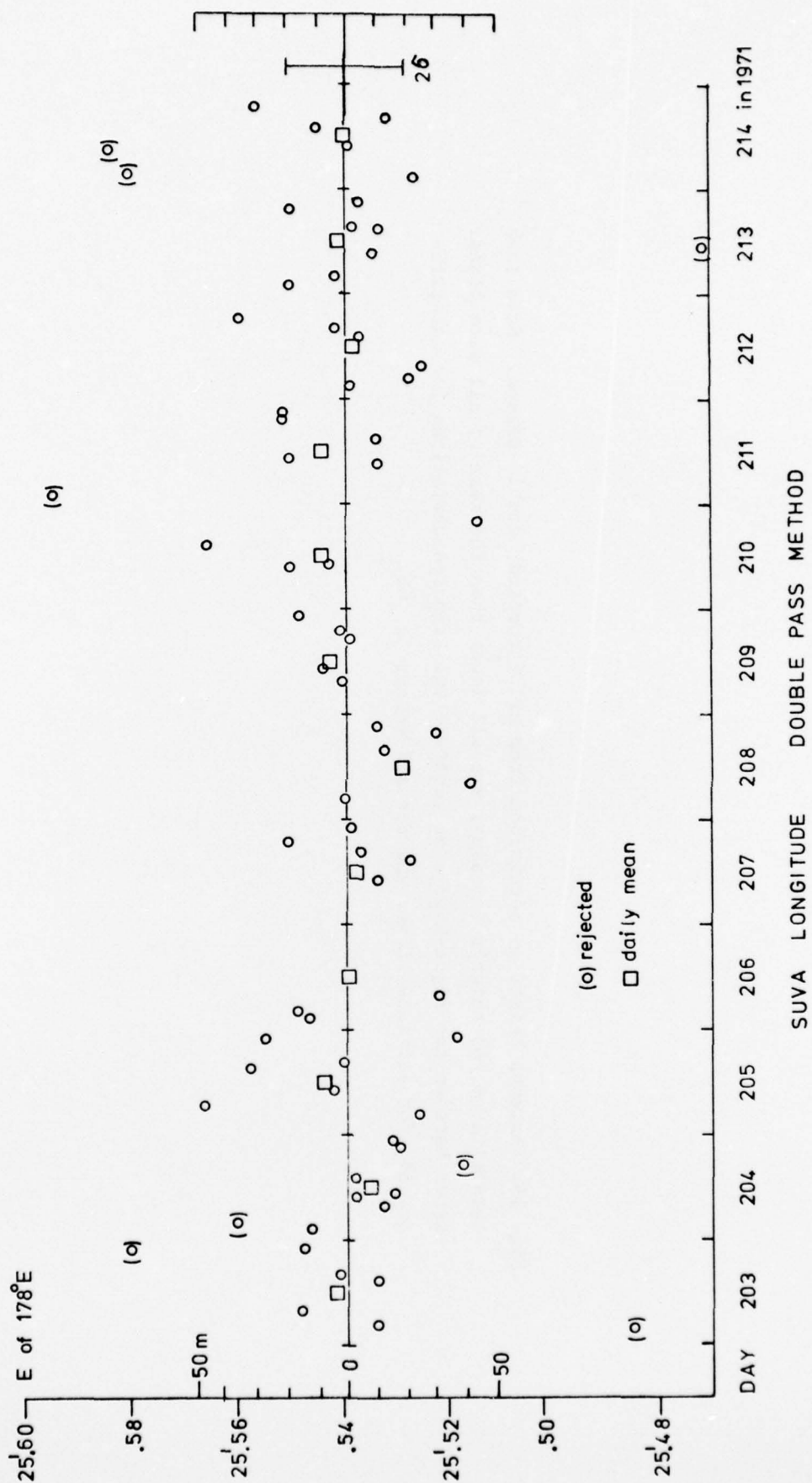


FIG.15

Fig. 16. Antenna height as determined from all (accepted) double passes. Note that satellite no. 65 heights (crosses) are all lower than the mean of all satellites. Error bars on the right-hand side indicate two standard deviations for satellite no. 65, for all satellites, and for all except no. 65.



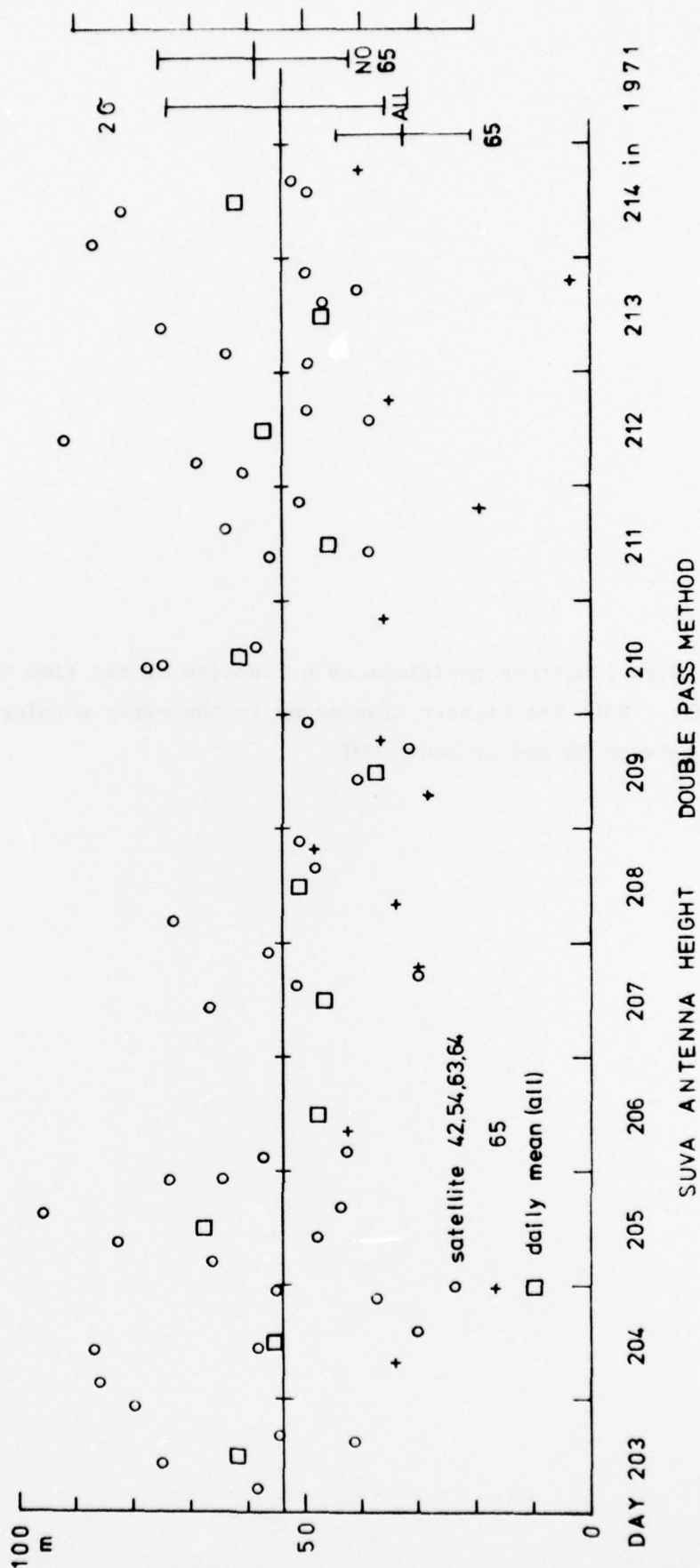


FIG.16

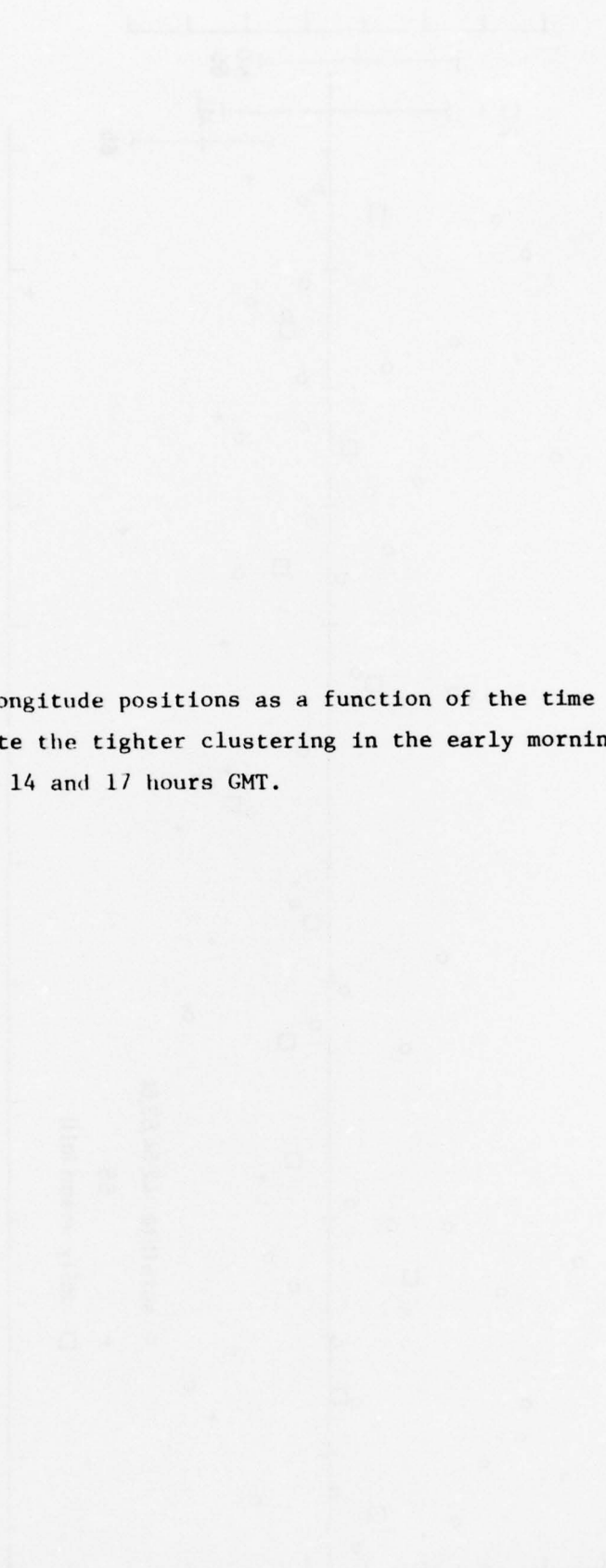


Fig. 17. Suva longitude positions as a function of the time of day (in GMT). Note the tighter clustering in the early morning (local) hours between 14 and 17 hours GMT.

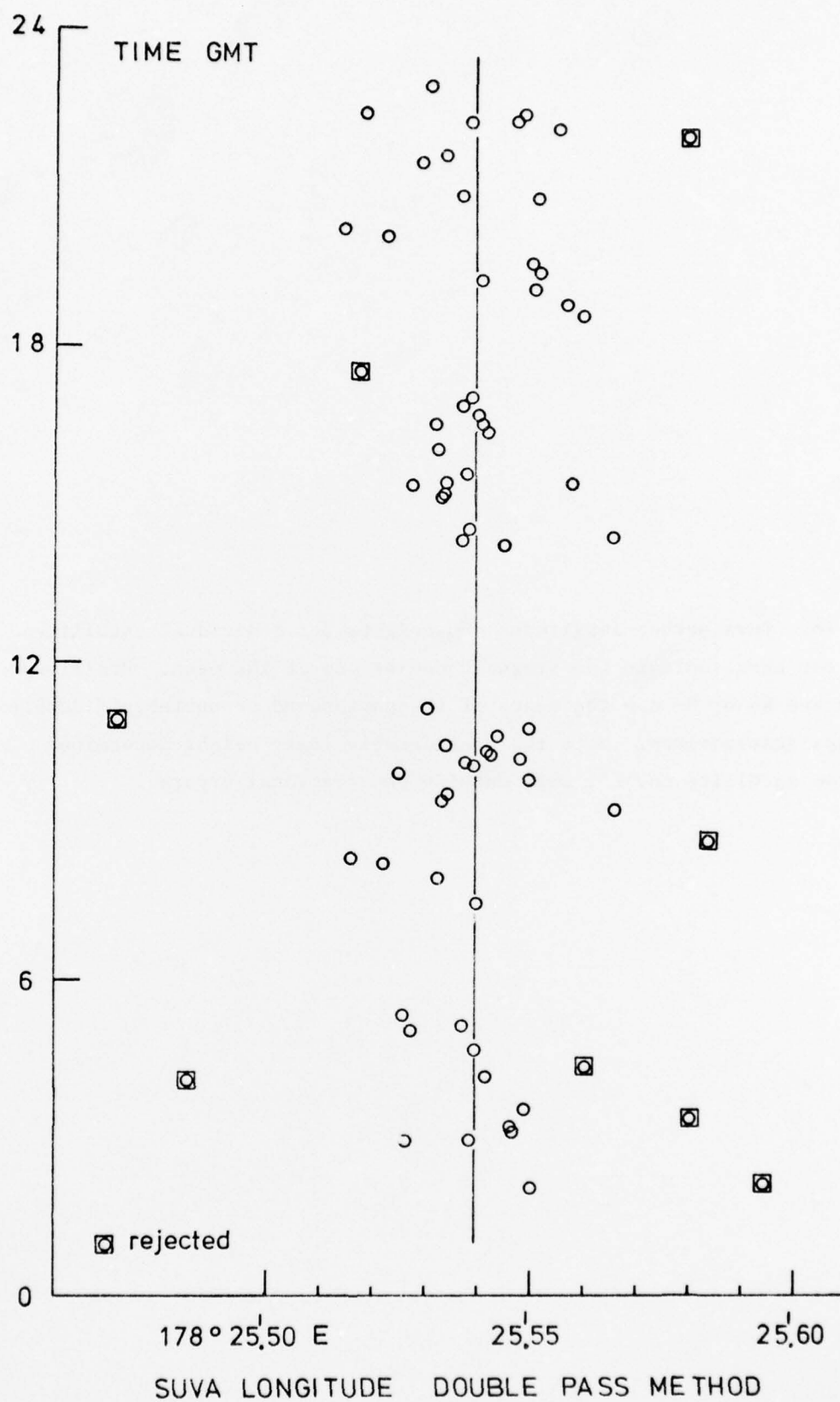
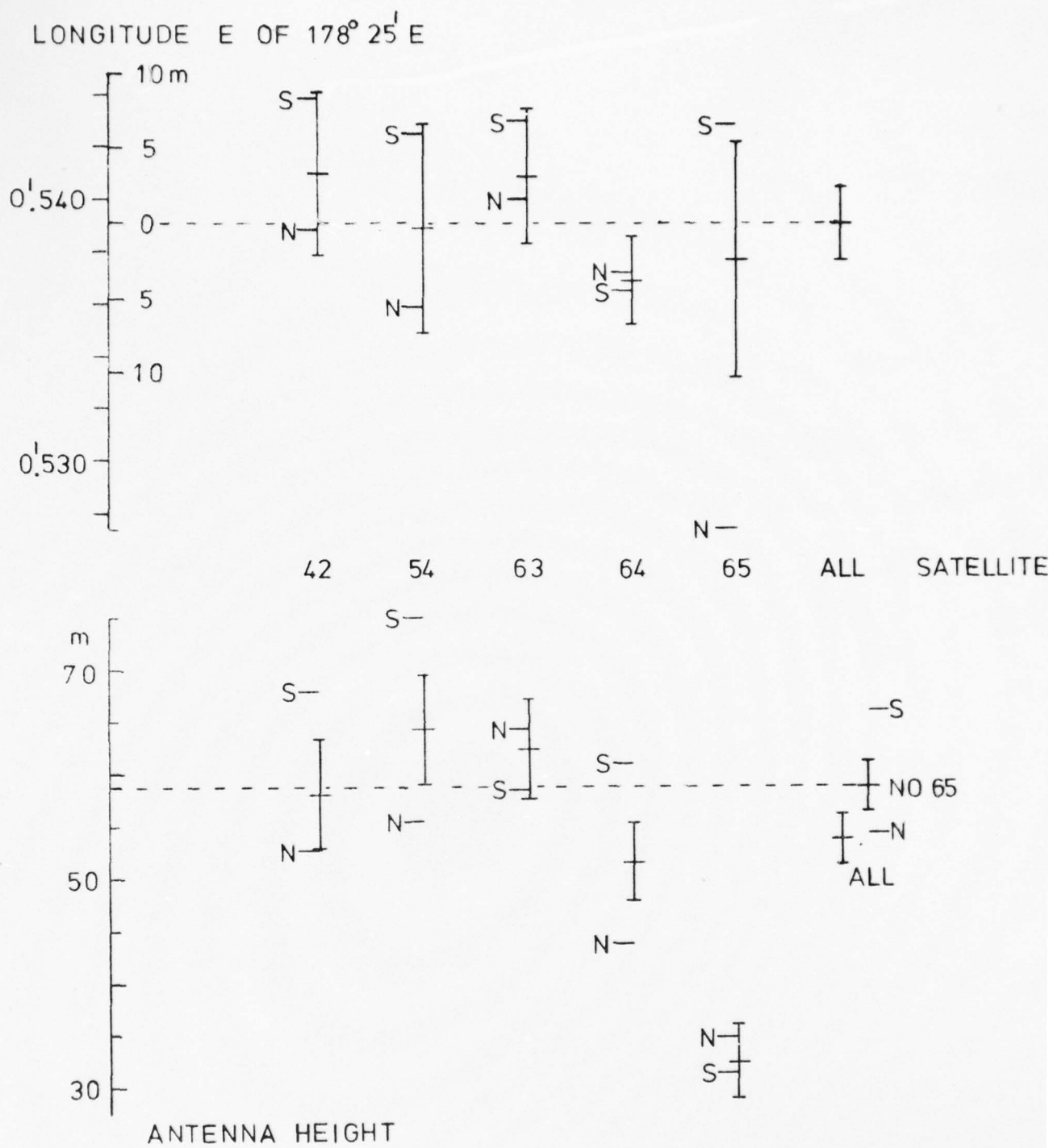


FIG.17

Fig. 18. Suva harbor longitudes and heights for individual satellites. Error bars indicate two standard deviations of the mean. Positions marked N- or S- are the means of the northbound or southbound double pass observations. Note the considerably lower height determined from satellite no. 65, well outside observational errors.





SUVA LOCATION 1971

FIG.18

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER HIG-75-11	2. GOVT ACCESSION NO. Geoidal	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) THE DOUBLE PASS METHOD FOR MORE PRECISE DETERMINATIONS OF LONGITUDE AND GEODAL HEIGHT USING THE TRANSIT SYSTEM OF NAVIGATIONAL SATELLITES		5. TYPE OF REPORT & PERIOD COVERED
7. AUTHOR 10 Eduard Berg		6. PERFORMING ORG. REPORT NUMBER HIG-75-11
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Using the Stansell (1970) elevation correction for navigational satellites together with the double pass method of Anderle (1971) and Greely (1971) but for individual navigational satellites on two consecutive passes, it is shown that uncertainties in longitude position can be reduced to the order of 10 to 18 m for land stations with just a few observations, and less than 4 m for dock-side ship stations taken over a (cont.)		

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period of a week. It is also shown that the method allows identification of satellites not giving reliable data and permits conclusions to be reached as to the cause of an apparent error dependance on surface elevation noted above 1000 m in the Andean region of Colombia, South America, reported by Woollard and Thompson (1974). An additional advantage of the double pass method brought out is that it allows geoidal heights to be determined for remote islands with the same degree of reliability as their position and comparable to the best, continental determinations of geoidal height using standard geodetic measurements.

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